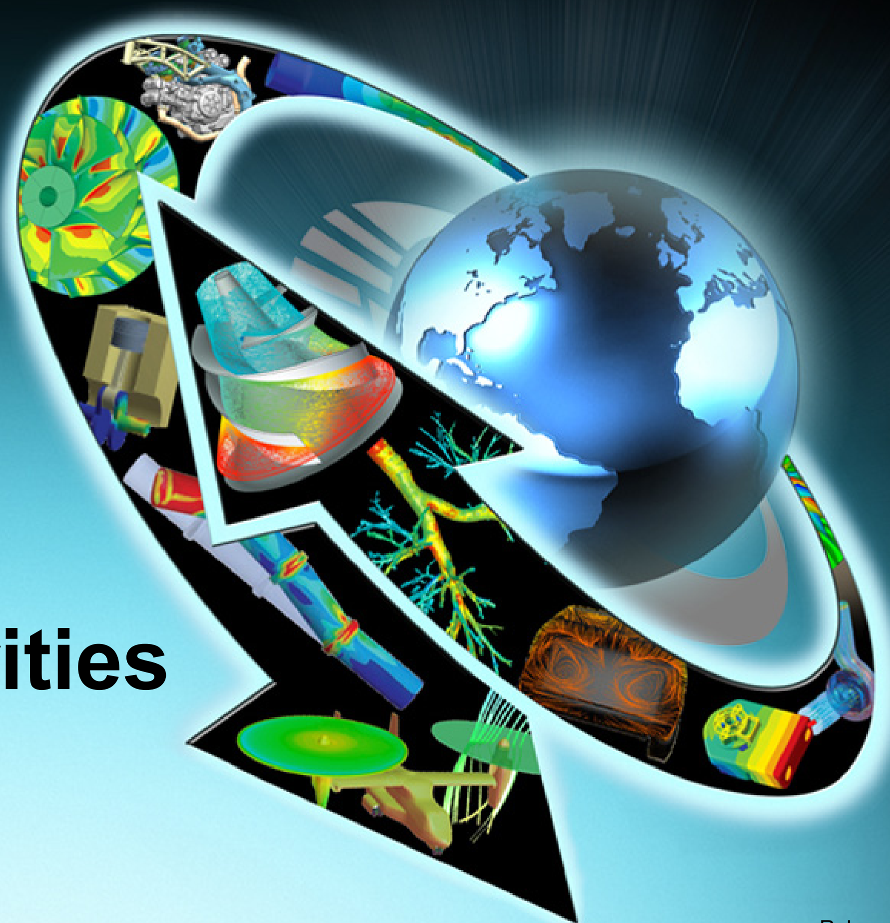
Customer Training Material

Lecture 5

Rate Independent Plasticity

ANSYS Mechanical Structural Nonlinearities



Release 13.0
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Chapter Overview

- **The following will be covered in this Chapter:**
  - **A. Background Elasticity/Plasticity**
  - **B. Yield Criteria**
  - **C. Hardening Rules**
  - **D. Material Data Input**
  - **E. Analysis Settings**
  - **F. Reviewing Results**
  - **G. Workshop**

- **The capabilities described in this section are generally applicable to *ANSYS Structural* licenses and above.**

A. Metal Plasticity Overview

Review of Elasticity:

- **Before proceeding to a discussion on plasticity, it may be useful to review elasticity of metals.**
  - **In elastic response, if the induced stresses are below the material's yield strength, the material can fully recover its original shape upon unloading.**
  - **From a standpoint of metals, this behavior is due to the stretching but not breaking of chemical bonds between atoms. Because elasticity is due to this stretching of atomic bonds, it is fully recoverable. Moreover, these elastic strains tend to be small.**
  - **Elastic behavior of metals is most commonly described by the stress-strain relationship of Hooke's Law:**

$$\sigma = E\varepsilon$$

… Metal Plasticity Overview

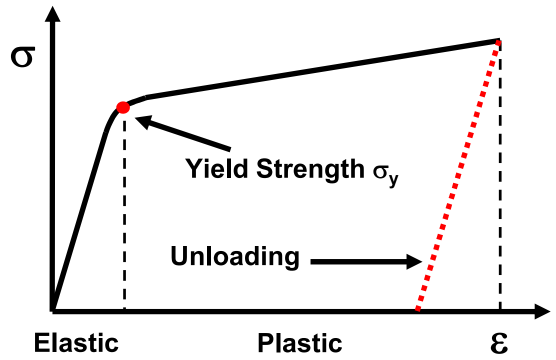
What is plasticity?

- **When a ductile material experiences stresses beyond the elastic limit, it will yield, acquiring large permanent deformations.**
  - **Plasticity refers to the material response beyond yield.**
  - **Plastic response is important for metal forming operations.**
  - **Plasticity is also important as an energy-absorbing mechanism for structures in service.**
    - **Materials that fail with little plastic deformation are said to be brittle.**
    - **Ductile response is safer in many respects than is brittle response.**

- **This Chapter will review some basics of plasticity by defining certain terminology.**

... Metal Plasticity Overview

- Plastic deformation results from slip between planes of atoms due to shear stresses (deviatoric stresses). This dislocation motion is essentially atoms in the crystal structure rearranging themselves to have new neighbors
  - results in unrecoverable strains or permanent deformation after load is removed.
  - slipping does not generally result in any volumetric strains (condition of incompressibility), unlike elasticity



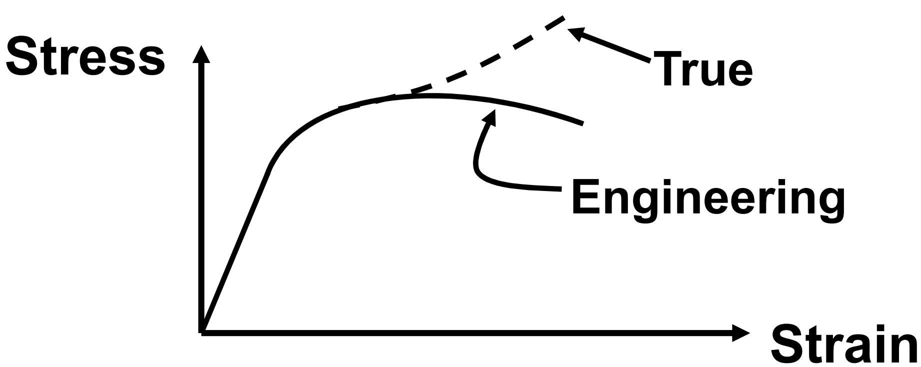
... Metal Plasticity Overview

Rate-Independent Plasticity:

- If the material response is not dependent on the rate of loading or deformation, the material is said to be *rate-independent*.
  - Most metals exhibit rate-independent behavior at low temperatures (< 1/4 or 1/3 melting temperature) and low strain rates.

Engineering vs. True Stress-Strain:

- While engineering stress-strain can be used for small-strain analyses, true stress-strain must be used for plasticity, as they are more representative measures of the state of the material.



Engineering vs. True Stress-Strain (cont'd):

- If presented with engineering stress-strain data, one can convert these values to true stress-strain with the following approximations:
  - Up until twice the strain at which yielding occurs:

$$\sigma = \sigma_{eng} \qquad \varepsilon = \varepsilon_{eng}$$

  - Up until the point at which necking occurs:

$$\sigma = \sigma_{eng}\left(1 + \varepsilon_{eng}\right) \qquad \varepsilon = \ln\left(1 + \varepsilon_{eng}\right)$$

  - Note that, only for stress conversion, the following is assumed:
    - Material is incompressible (acceptable approximation for large strains)
    - Stress distribution across cross-section of specimen is assumed to be uniform.
  - Beyond necking:
    - There is no conversion equation relating engineering to true stress-strain at necking. The instantaneous cross-section must be measured.

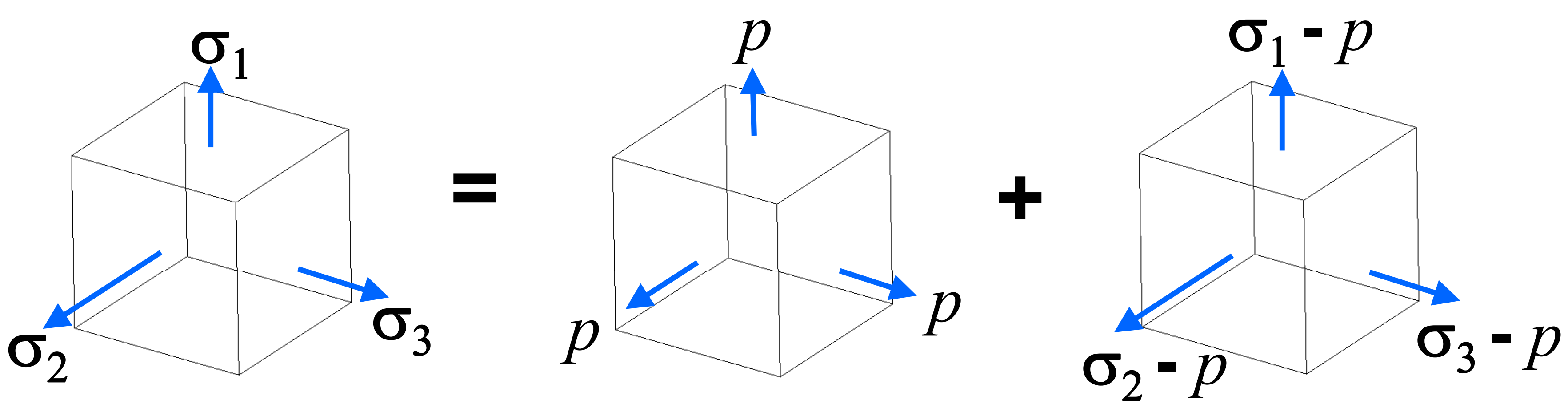
B. Yield Criterion

Yield Criterion:

- **The yield criteria is used to relate multiaxial stress state with the uniaxial case.**
  - **Tensile testing on specimens provide uniaxial data, which can easily be plotted on one-dimensional stress-strain curves, such as those presented earlier in this section.**
  - **The actual structure usually exhibits multiaxial stress state. The yield criterion provides a scalar invariant measure of the stress state of the material which can be compared with the uniaxial case.**

... Yield Criterion

- **In general, a stress state can be separated into two components.**
  - **Hydrostatic stress - generates volume change.**
  - **Deviatoric stress - generates angular distortion.**



Stress State
(Where: $\sigma_1 \neq \sigma_2 \neq \sigma_3$)

Hydrostatic stress (p) causing volume change only

Deviatoric stress causing angular distortion only

… Yield Criterion

- **The von Mises yield criterion predicts that yielding will occur whenever the distortion energy in a unit volume equals the distortion energy in the same volume when uniaxially stressed to the yield strength.**
  - **From this theory, a scalar invariant (von Mises equivalent stress) is derived as:**

$$\sigma_e = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

- **When von Mises equivalent stress exceeds the uniaxial material yield strength, general yielding will occur.**

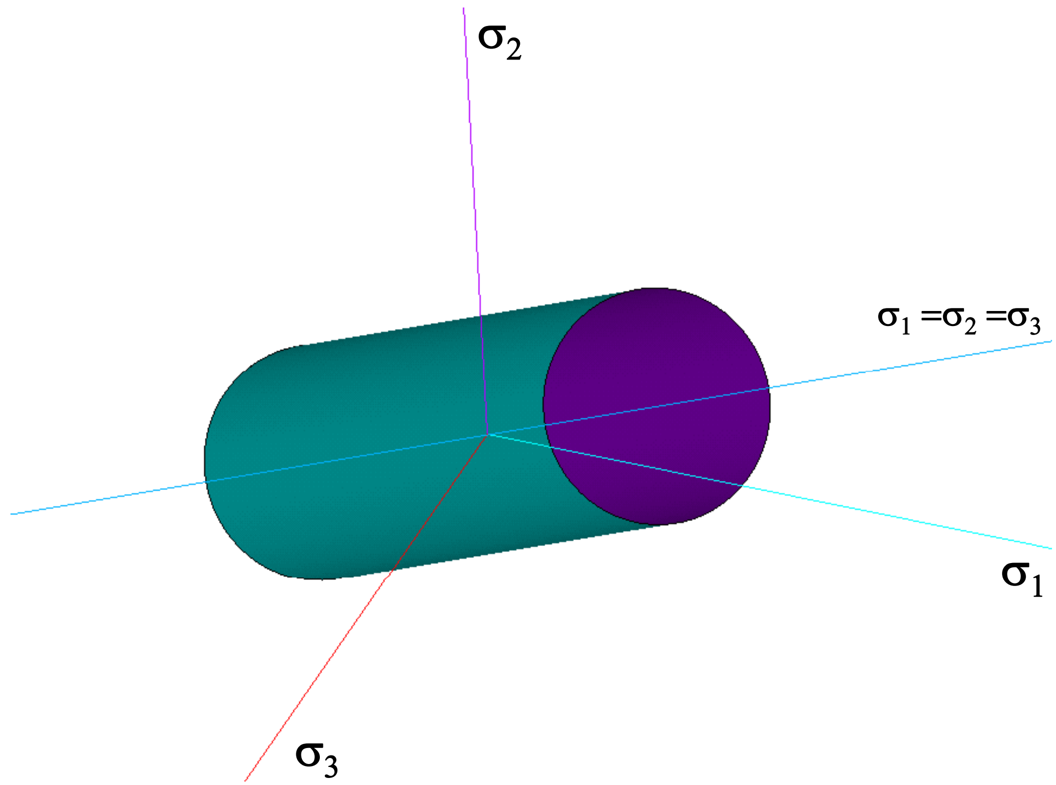
... Yield Criterion

- **If plotted in 3D principal stress space, the von Mises yield surface is a cylinder.**

The cylinder is aligned with the axis $\sigma_1=\sigma_2=\sigma_3$.

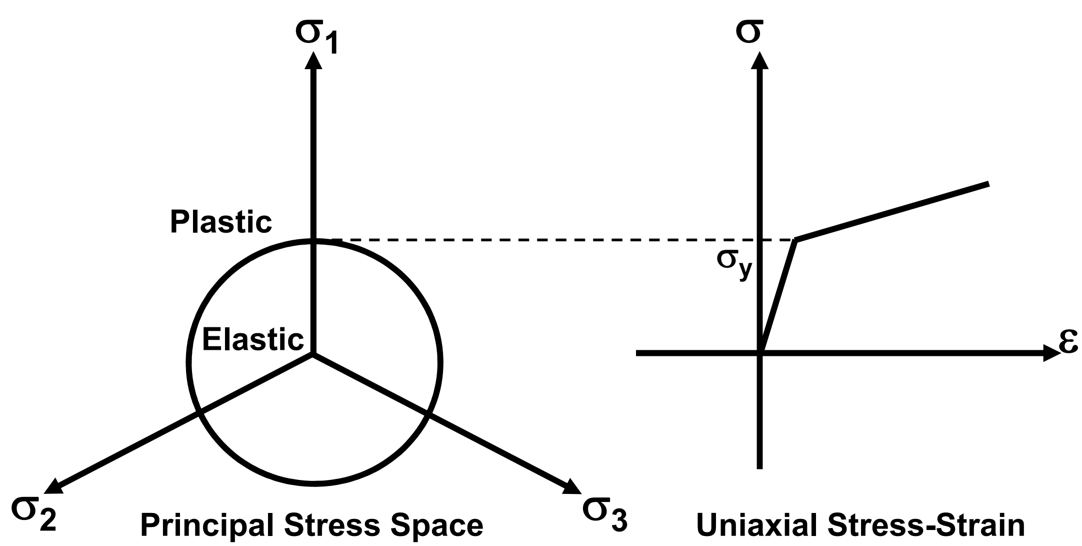
Note that if the stress state is inside the cylinder, no yielding occurs. This means that if the material is under hydrostatic pressure ($\sigma_1=\sigma_2=\sigma_3$), no amount of hydrostatic pressure will cause yielding.

Another way to view this is that stresses which deviate from the axis ($\sigma_1=\sigma_2=\sigma_3$) contribute to the von Mises stress calculation.



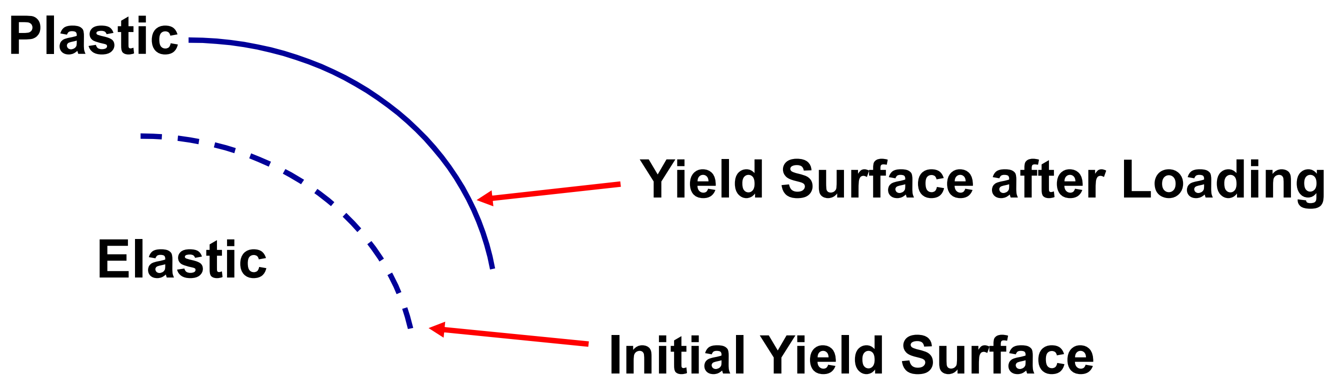
… Yield Criterion

- **At the edge of the cylinder (circle), yielding will occur.**
- **No stress state can exist *outside* of the cylinder.**
- **Instead, hardening rules will describe how the cylinder changes with respect to yielding.**



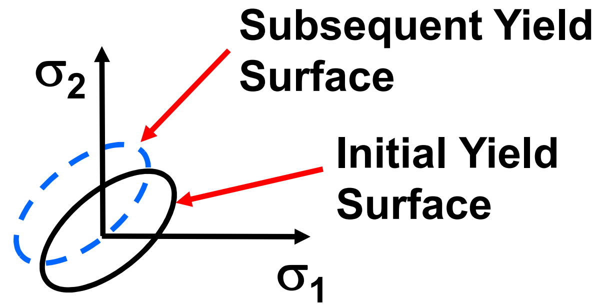
C. Hardening Rules

- The hardening rule describes how the yield surface changes (size, center,shape) as the result of plastic deformation.
- The hardening rule determines when the material will yield again if the loading is continued or reversed.
  - This is in contrast to elastic-perfectly-plastic materials which exhibit no hardening -- i.e., the yield surface remains fixed.

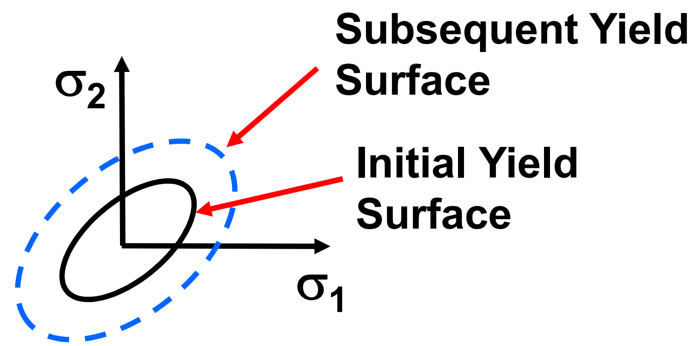


… Hardening Rules

- There are two basic hardening rules to prescribe the modification of the yield surface:
  - *Kinematic* hardening.
    - The yield surface remains constant in size and translates in the direction of yielding.



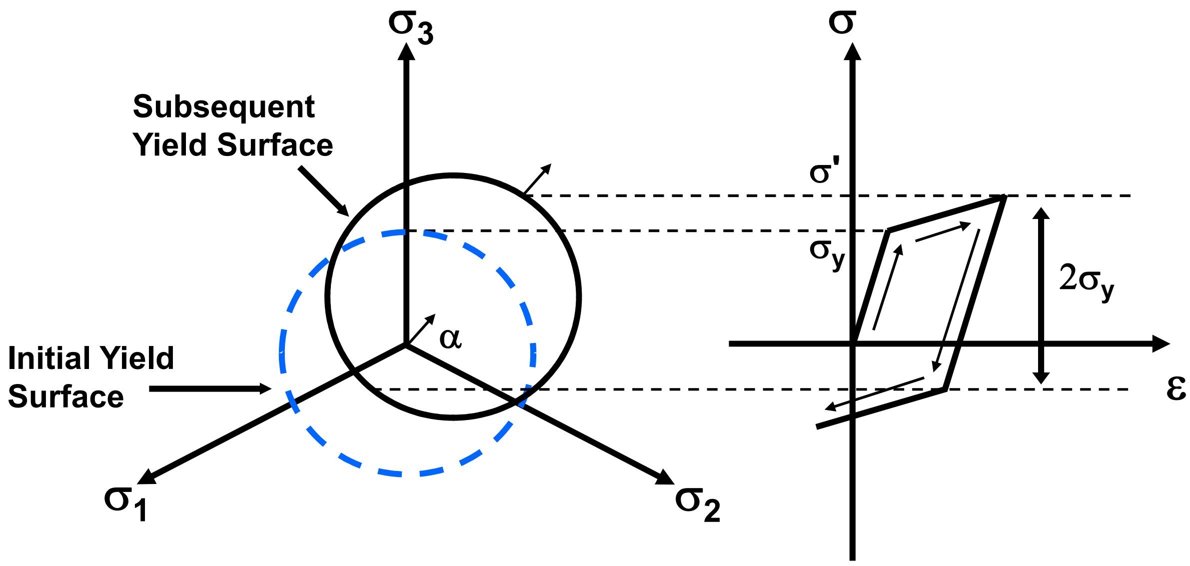
  - *Isotropic* hardening.
    - The yield surface expands uniformly in all directions with plastic flow.



- Most metals exhibit kinematic hardening behavior for small strain cyclic loading.

… Kinematic Hardening

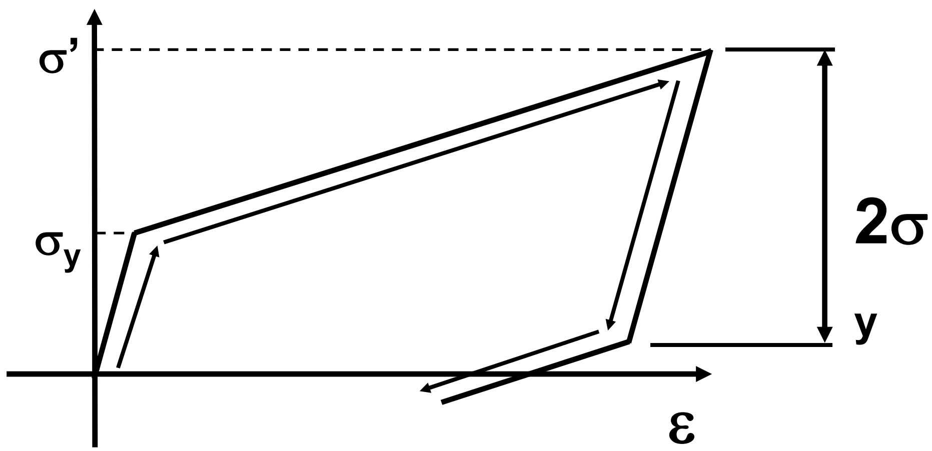
- The stress-strain behavior for linear kinematic hardening is illustrated below:



- Subsequent yield in compression is decreased by the amount that the yield stress in tension increased, so that a $2\sigma_y$ difference between the yields is always maintained. (This is known as the *Bauschinger effect.*)

… Kinematic Hardening

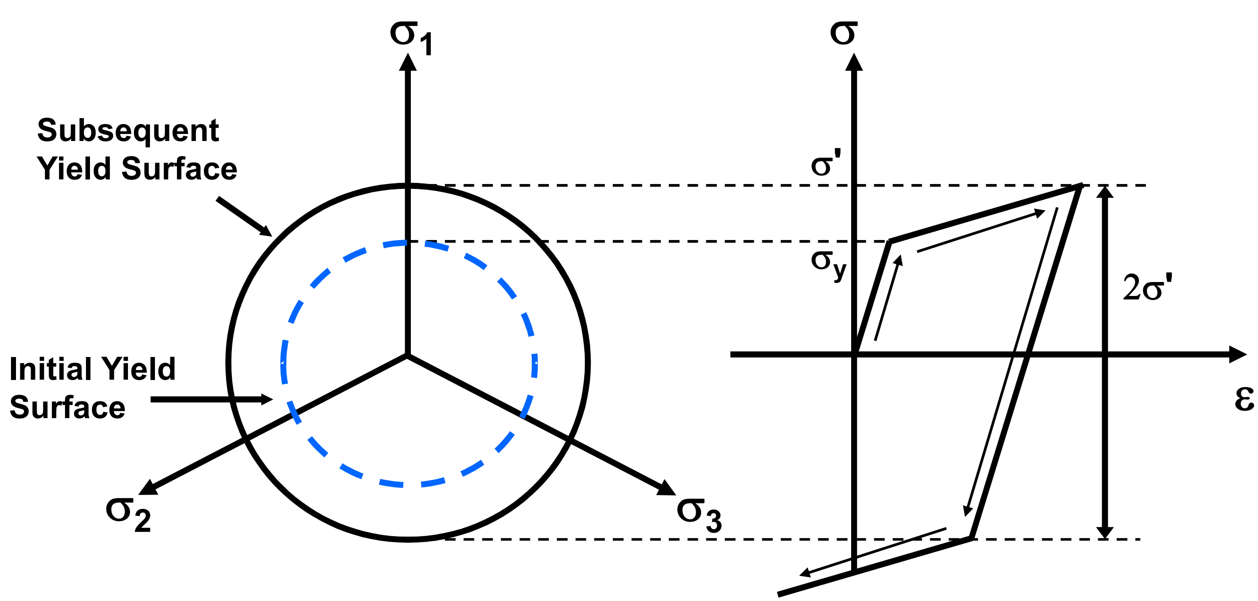
- An initially isotropic material is no longer isotropic after it yields and experiences kinematic hardening.
- For very large strain simulations, the linear kinematic hardening model can become inappropriate because of the Bauschinger effect.



- Kinematic hardening is generally used for small strain, cyclic loading applications.

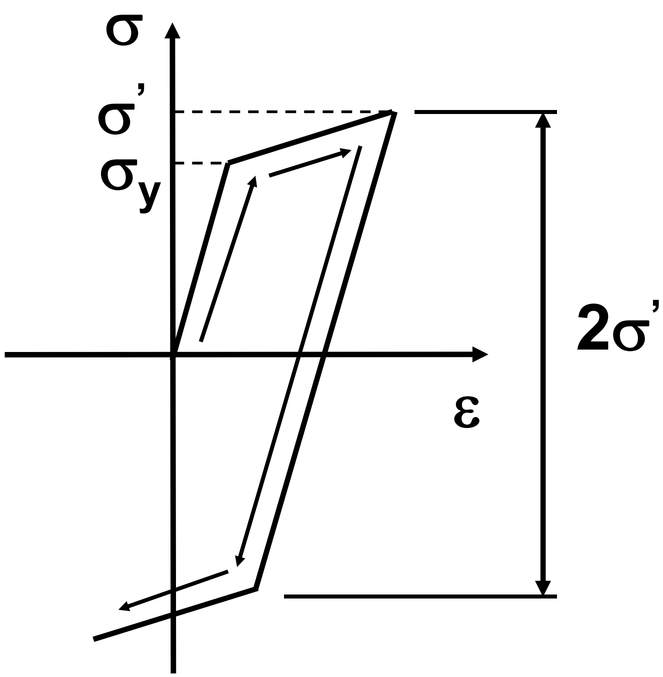
… Isotropic Hardening

- **Isotropic hardening states that the yield surface expands uniformly during plastic flow. The term 'isotropic' refers to the uniform dilatation of the yield surface and is different from an 'isotropic' yield criterion (i.e., material orientation).**



… Isotropic Hardening

- **Plotting the stress-strain curve enables an understanding of what occurs during a loading and reverse loading cycle:**



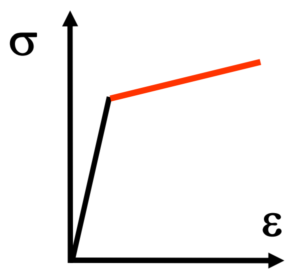
Note that the subsequent yield in compression is equal to the highest stress attained during the tensile phase.

Isotropic hardening is often used for large strain or proportional loading simulations. It is usually not applicable for cyclic loading.

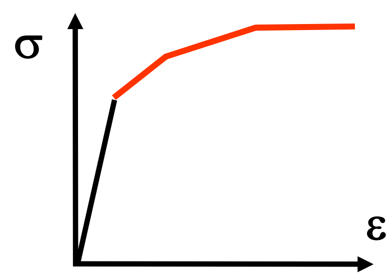
D. Material Data Input

Curve shapes

- **Two different type of stress-strain curve representations are possible:**



Bilinear



Multilinear

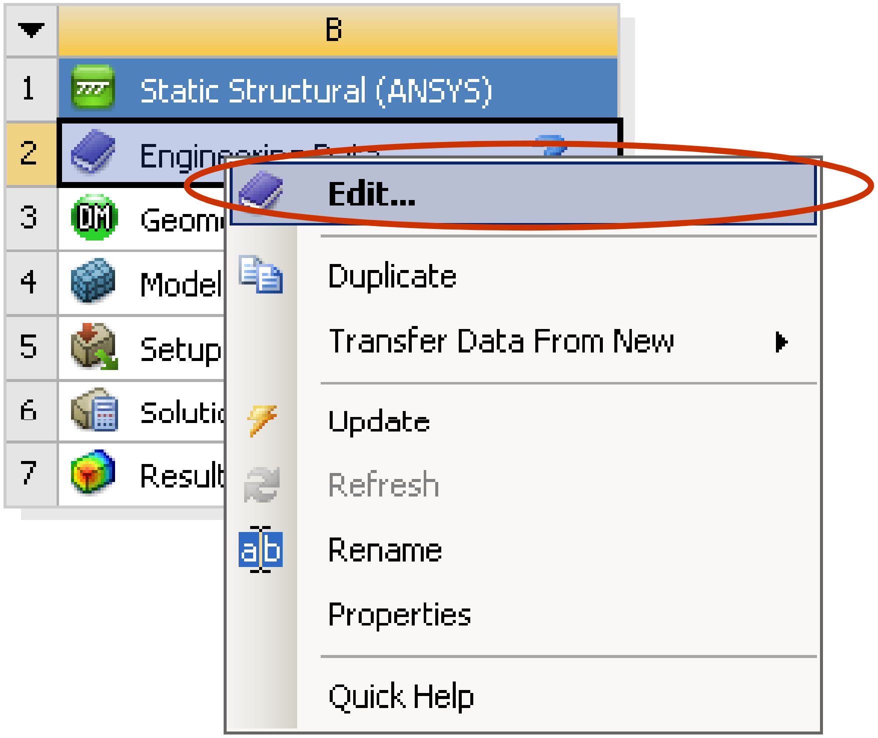
... Material Data Input

- **Linear elastic material properties must be supplied**
  - **The same requirements exist for linear static structural analyses, namely that Young's Modulus and Poisson's Ratio must be defined as a minimum.**

- **Metal plasticity is available as a nonlinear material model. This will be discussed next.**
  - **Other nonlinear constitutive models may be added with *Command Objects***
  - **Note that only *ANSYS Professional* NLS licenses and above support nonlinear material laws.**

... Material Data Input

- **To add metal plasticity, first navigate to the project schematic. Highlight the Engineering Data branch, double click or RMB and click on Edit…**



... Material Data Input

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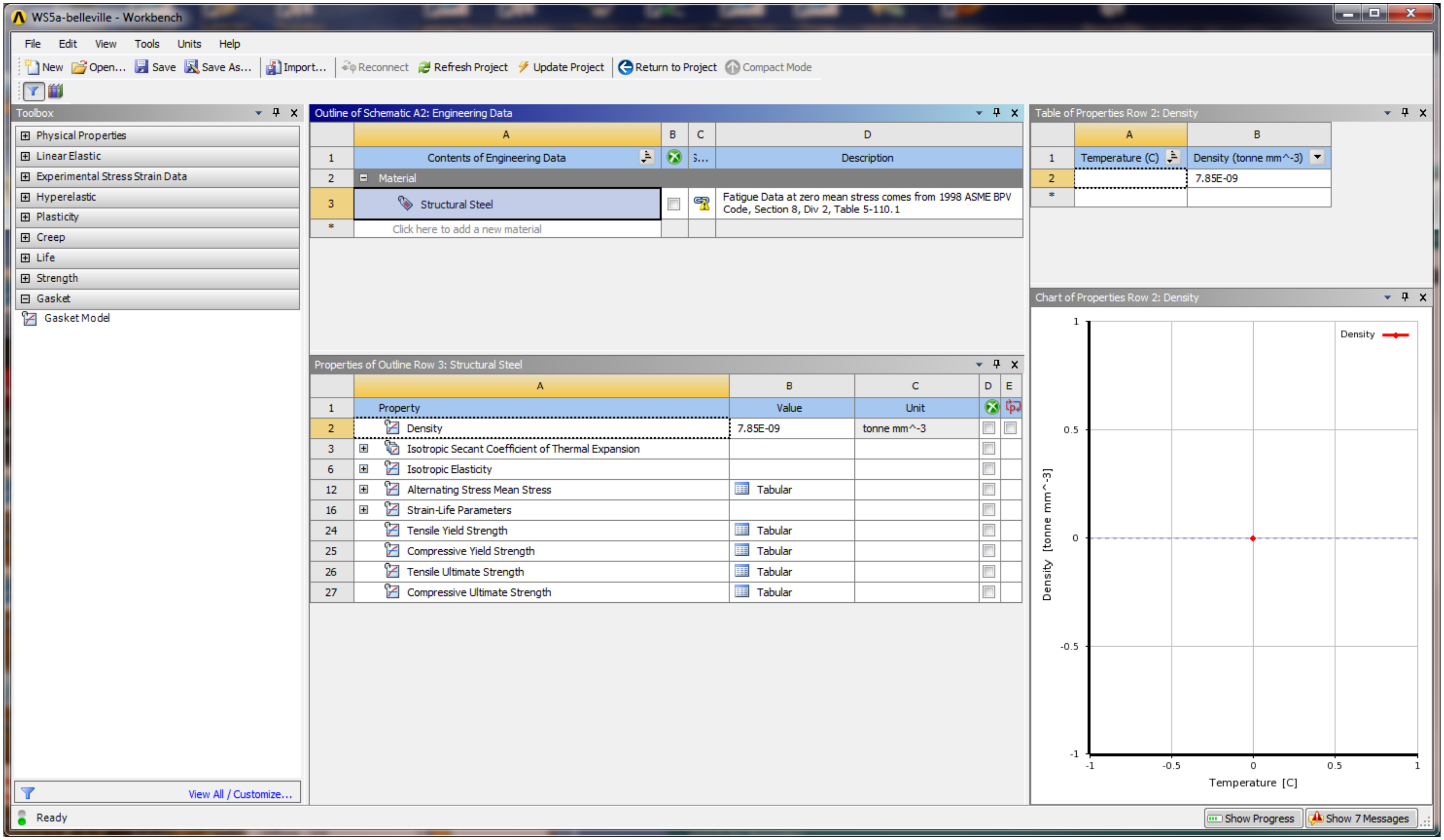
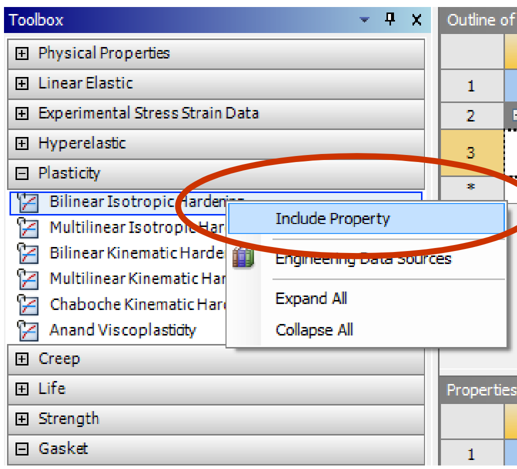
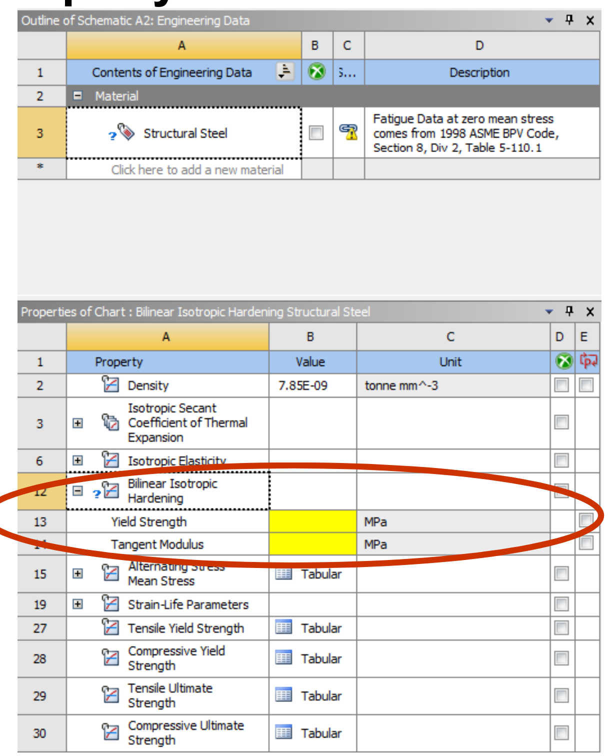
- **This opens the Engineering Data dialogue box for adding and editing various material properties related to the active project(s).**



... Material Data Input

- **From the Toolbox, open the plasticity folder:**
  - **Highlight the metal plasticity model of interests (in the example below, Bilinear Isotropic is selected)**
  - **RMB on the material model and click on "Include Property"**





  - **The Bilinear Isotropic Hardening model will then appear in the Properties Dialogue box.**
  - **The yellow blank boxes are now available for user to define yield strength and tangent modulus.**

... Material Data Input

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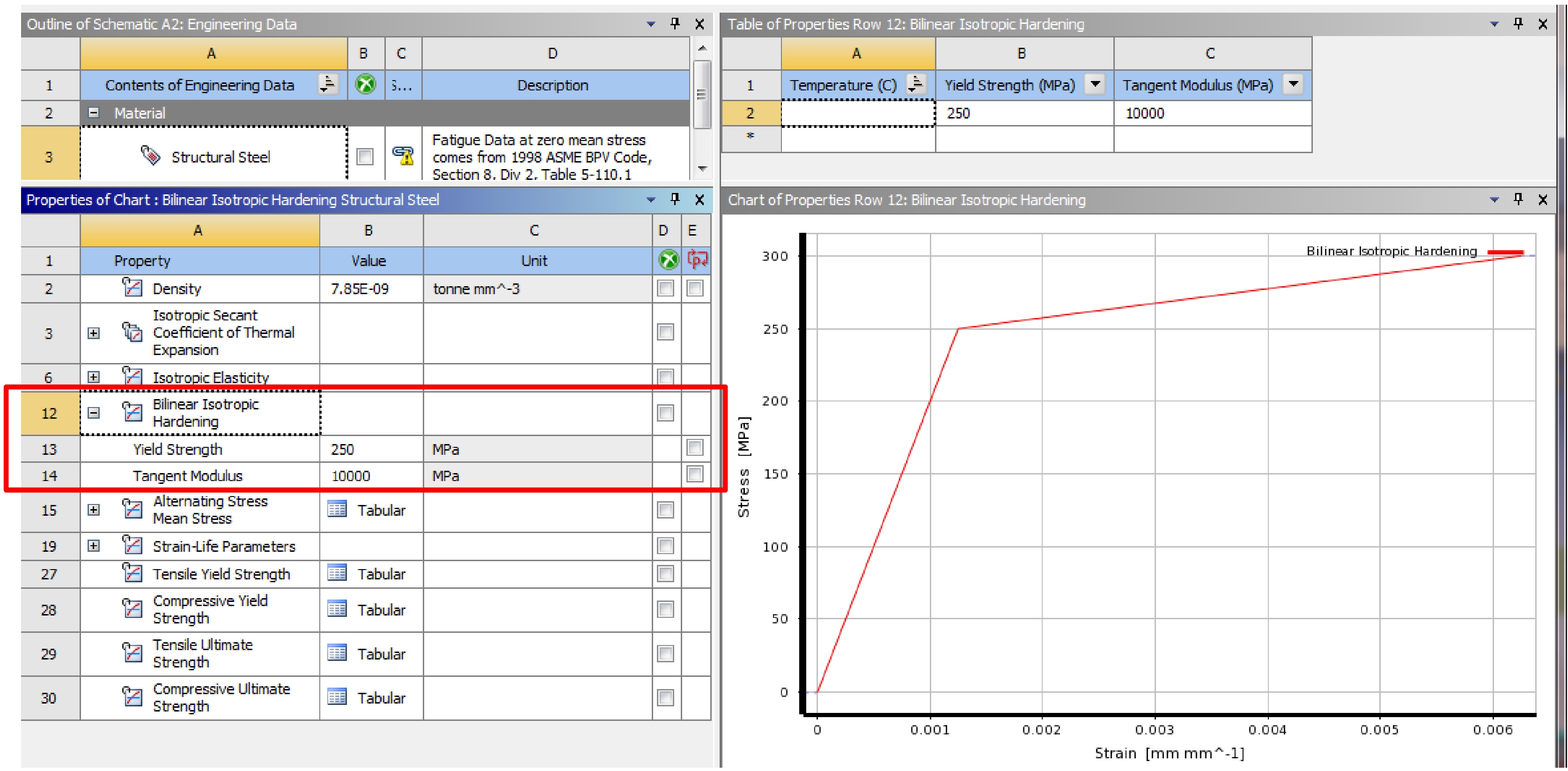
- **After defining the yield strength and tangent modulus, the data will automatically be plotted graphically for inspection:**



... Material Data Input

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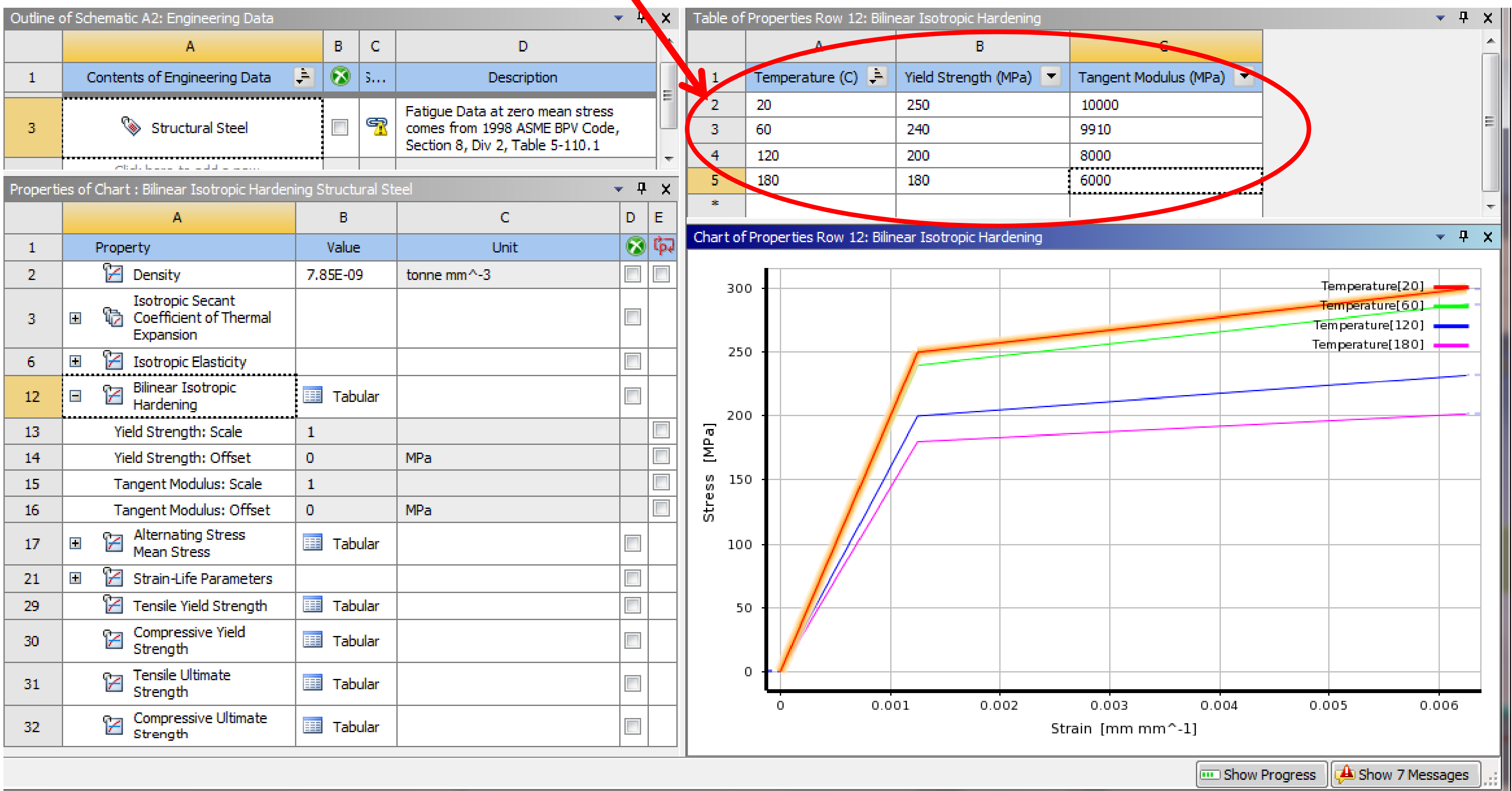
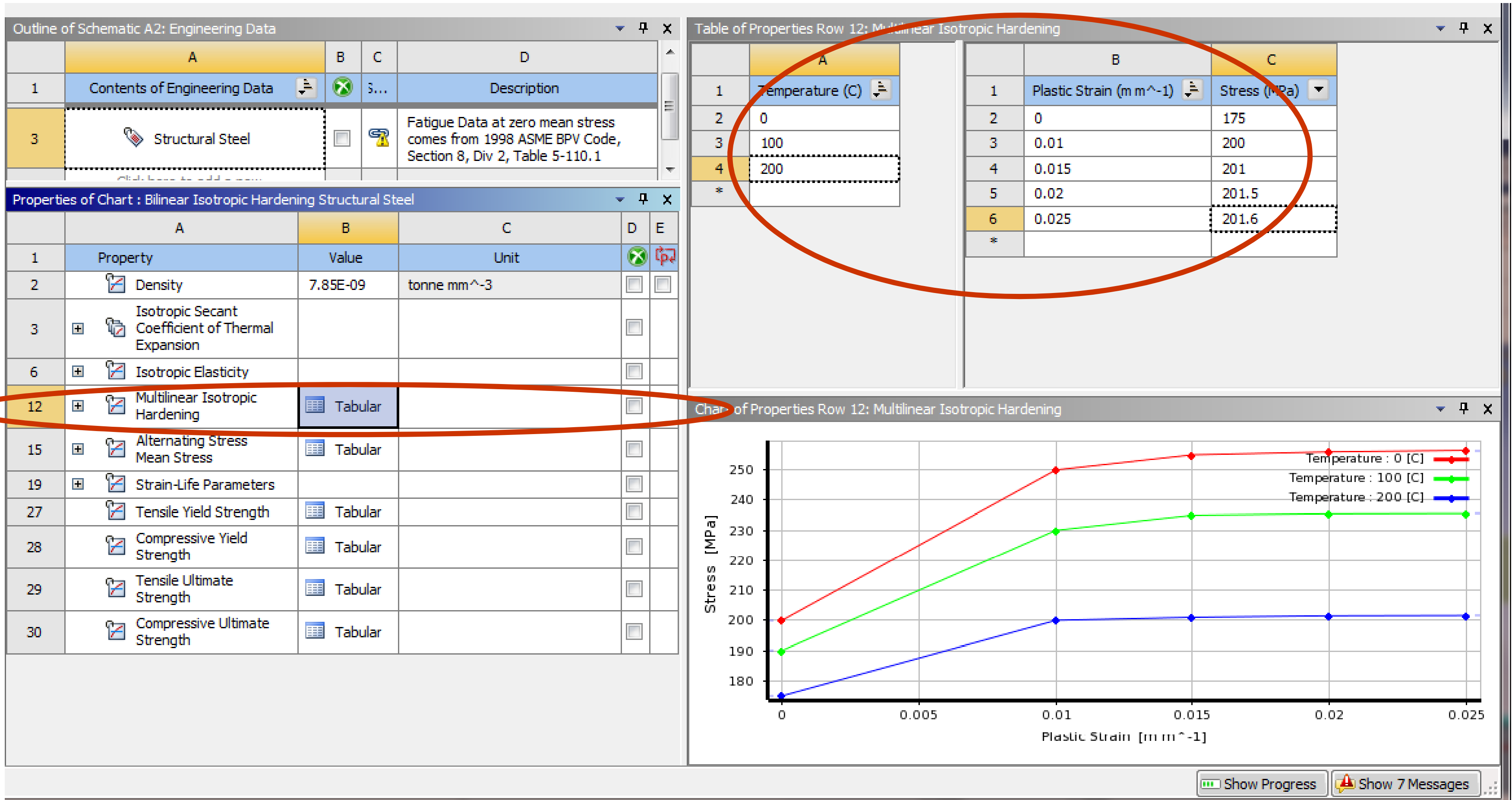
- **Bilinear isotropic or kinematic hardening models also support temperature dependent properties via Tabular input.**



... Material Data Input

- **In a similar procedure, multilinear isotropic or kinematic hardening models can also be defined and verified:**



E. Analysis Settings for Metal Plasticity

- **Ensure that the substep size is adequate to capture the path dependent response accurately with minimal bisections.**
  - **Solver will trigger a bisection automatically for plastic strains exceeding 15% in a substep**
  - **Refer to CUTCONTROL command doc.**
- **Large Deflection = ON is recommended**
- **For large models with long run times and potential convergence trouble, consider setting up a Restart Control strategy in the event that adjustment to time step range or convergence criteria is necessary**

Details of "Analysis Settings"

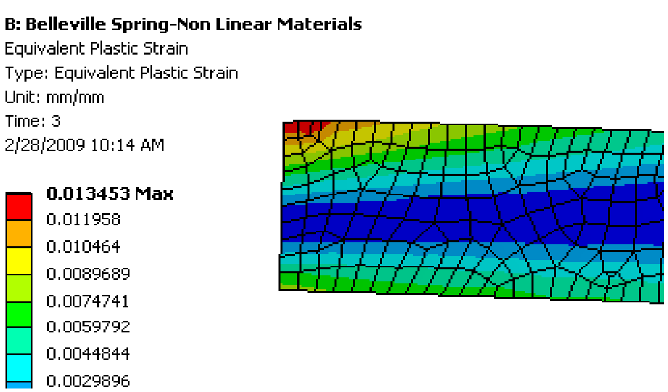
Step Controls	
Number Of Steps	1.
Current Step Number	1.
Step End Time	1. s
Auto Time Stepping	On
Define By	Substeps
Initial Substeps	25.
Minimum Substeps	5.
Maximum Substeps	100.
Solver Controls	
Solver Type	Program Controlled
Weak Springs	Off
Large Deflection	On
Inertia Relief	Off
Restart Controls	
Generate Restart Points	Manual
Load Step	All
Substep	Specified
Rate of Recurrence	5
Maximum Points to Save Per Step	All
Retain Files After Full Solve	No
Nonlinear Controls	
Force Convergence	Program Controlled
Moment Convergence	Program Controlled
Displacement Convergence	Program Controlled

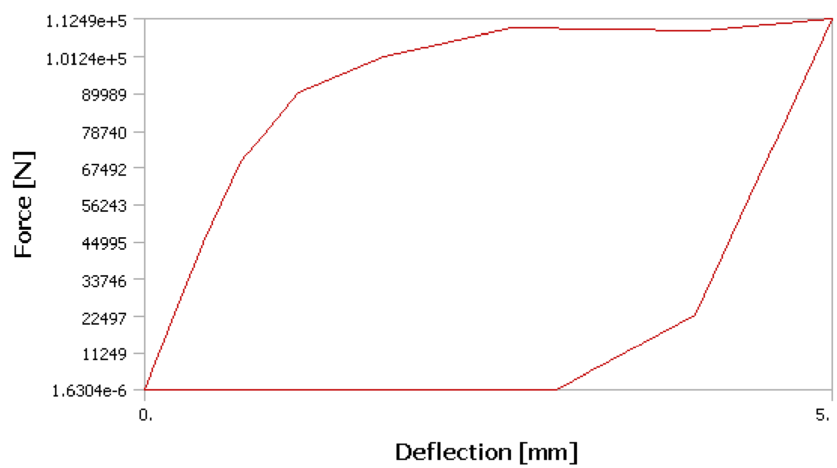
F. Reviewing Results

- **Reviewing results in a metal plasticity model is similar to a linear elastic run with the exception that there is now a path dependent plastic strain to consider.**
  - **Review multiple results sets along the path**

```
*** LOAD STEP     2   SUBSTEP     3  COMPLETED.    CUM ITER =     16
*** TIME =   1.08575         TIME INC =  0.367500E-01
*** MAX PLASTIC STRAIN STEP = 0.4841E-04   CRITERION = 0.1500
*** AUTO TIME STEP:  NEXT TIME INC = 0.55125E-01  INCREASED (FACTOR = 1.5000)
```

  - **Examine the nonlinear force deflection curve to better understand how the plastic strain is influencing the overall nonlinearity of the structure.**





… Summary of Plasticity in Mechanical

- **Metal plasticity deals with elastic and *inelastic* (permanent) deformation. Inelastic or plastic deformation occurs when the stress is higher than the *yield strength*. There will always be some recoverable strain (elastic strain) upon unloading.**

- **A stress-strain curve is based on *scalar* data, usually from a uniaxial test. A system may undergo a multiaxial stress state, so WB-Mechanical uses the *Mises yield criterion* to relate a multiaxial stress state with scalar test data. In this situation, *true stress vs. strain* data should be supplied.**

- **After yielding occurs, the yield point may increase due to *strain hardening*. This changes the *yield surface*, and the way in which it evolves in Simulation is determined by *Isotropic or Kinematic hardening* assumption.**

- **The stress-strain curve can be represented by a *bilinear* or *multilinear* curve.**

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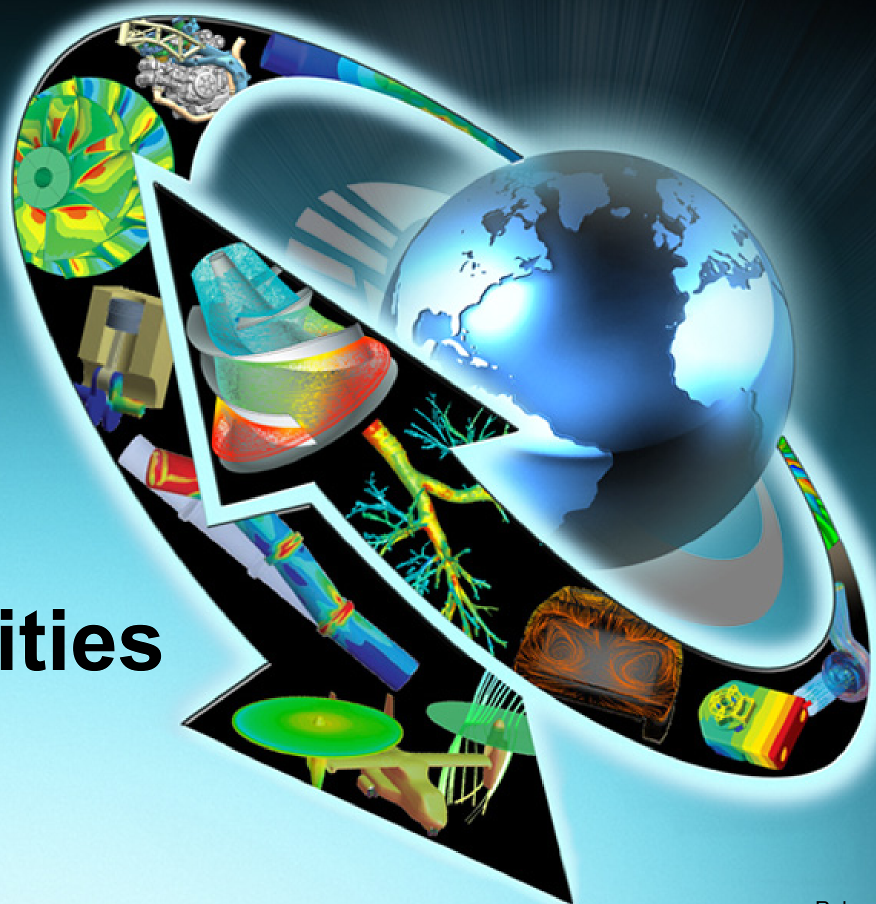
Workshop 5A

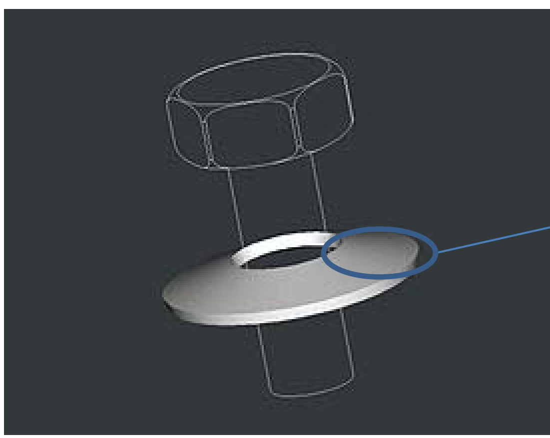
Metal Plasticity

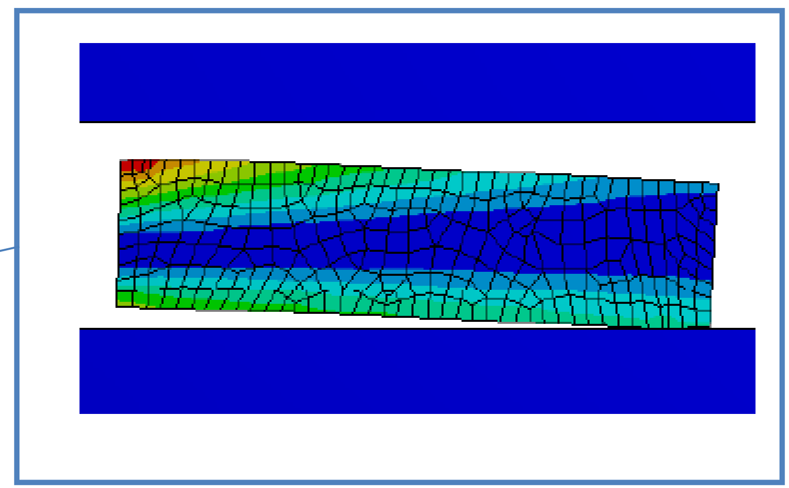


ANSYS Mechanical Structural Nonlinearities

Belleville Washer (Spring)

- A Belleville washer, also called a Belleville spring, is a cone shaped washer that deforms to preload a bolted joint
- The Belleville washer acts like a lock washer and also provides some compliance to a bolted joint

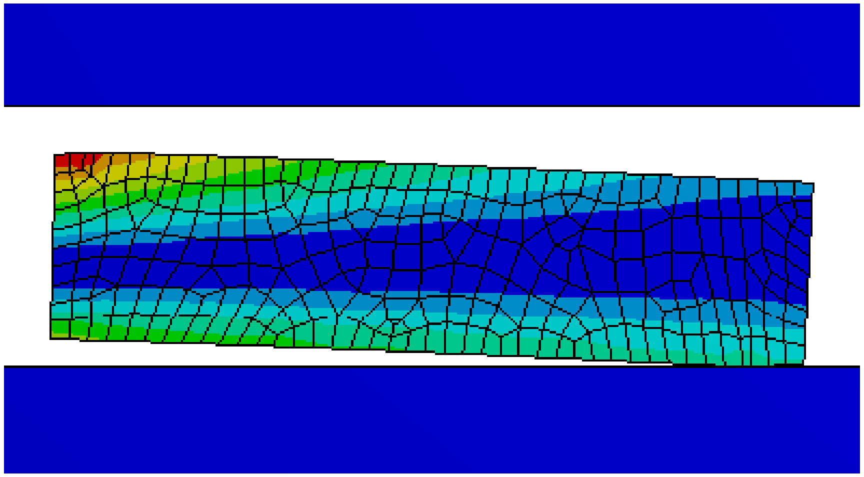


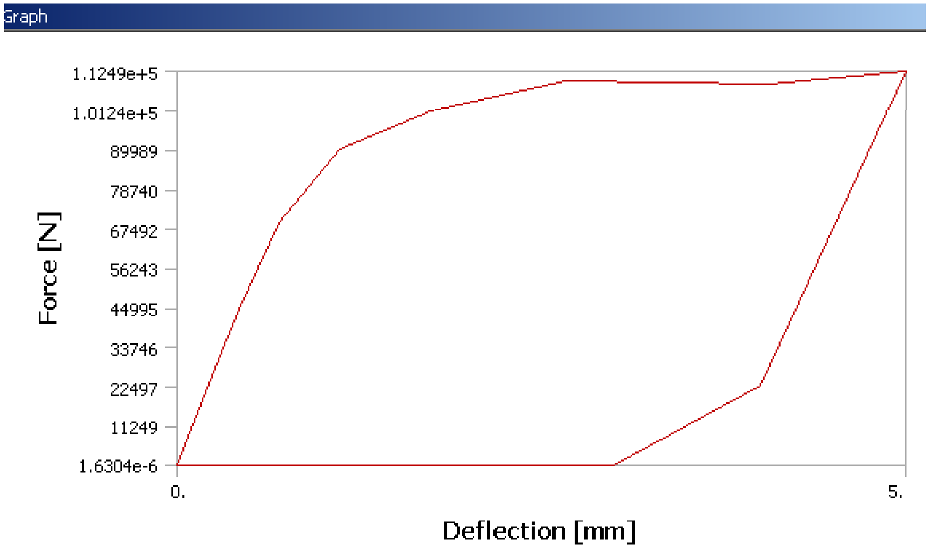


Axi-symmetric FE model of washer and contact surfaces

Workshop 5A – Metal Plasticity

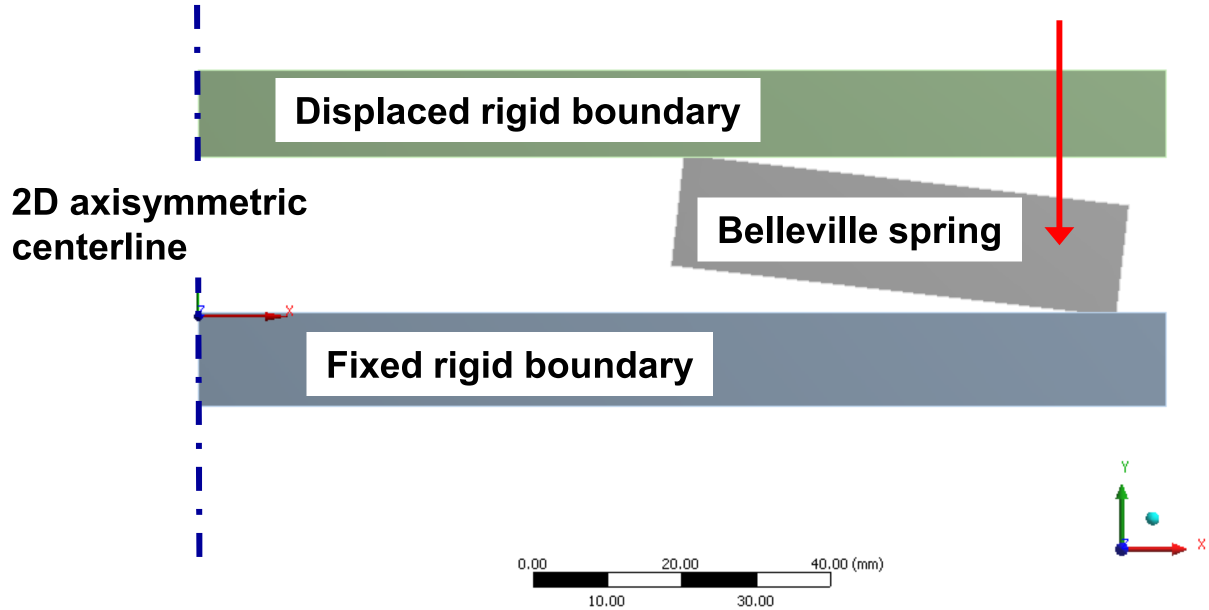
- **Goal:**
  - **Define a nonlinear metal plasticity material for a belleville spring geometry and simulate "spring back" upon application of and subsequent removal of a displacement load.**
  - **Post process stress and strain results**
  - **Generate a force vs. deflection curve on the spring.**





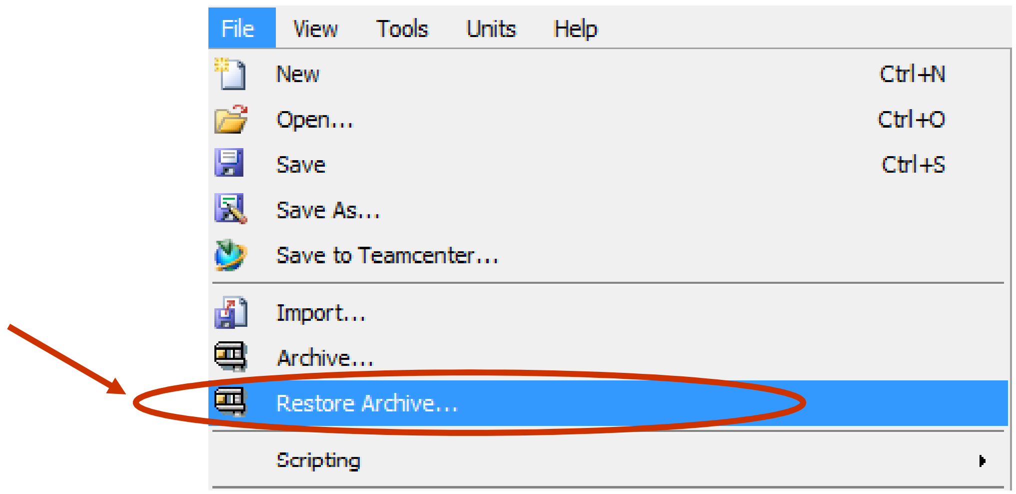
… Workshop 5A – Metal Plasticity

- 2D axisymmetric geometry
- The spring material is a ductile steel sandwitched between two rigid surfaces.
- Frictionless contact is assumed between the spring and the rigid geometries

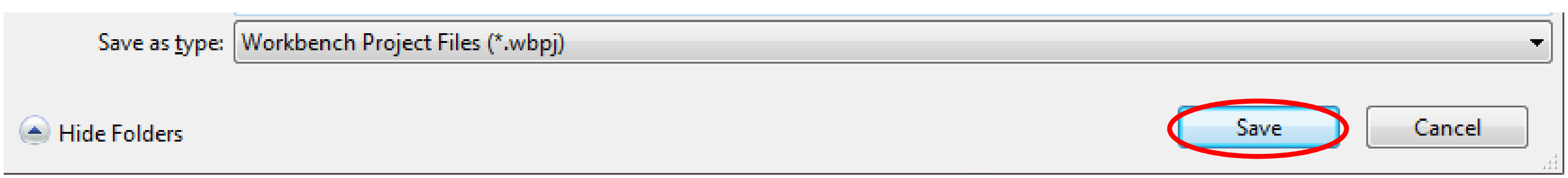


Steps to Follow:

- **Restore Archive… browse for file "W5a-belleville.wbpz"**



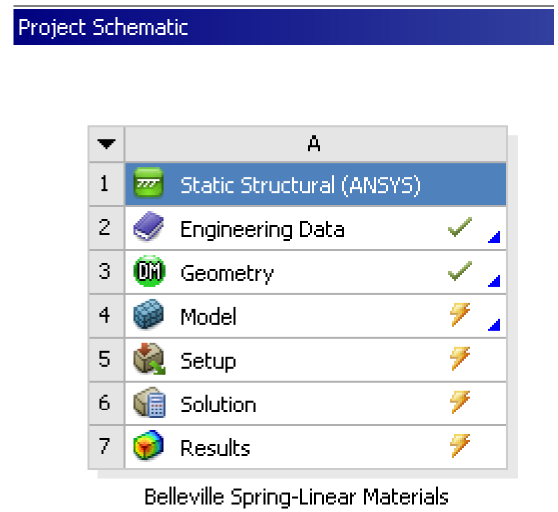
- **Save as**
  - **File name: "W5a-belleville"**
  - **Save as type: Workbench Project Files (*.wbpj)**



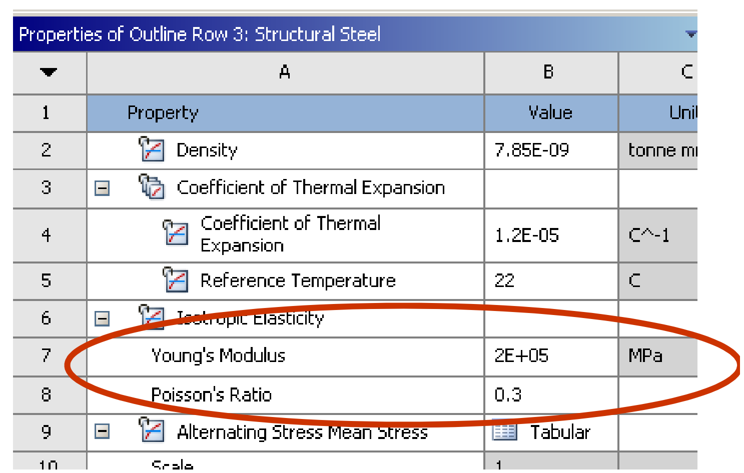
… Workshop 5A – Metal Plasticity

The project Schematic should look like the picture to the right.

- **From this Schematic, you can see that Engineering (material) Data and Geometry have already been defined (green check marks).**
- **It remains to set up and run the FE model in Mechanical**

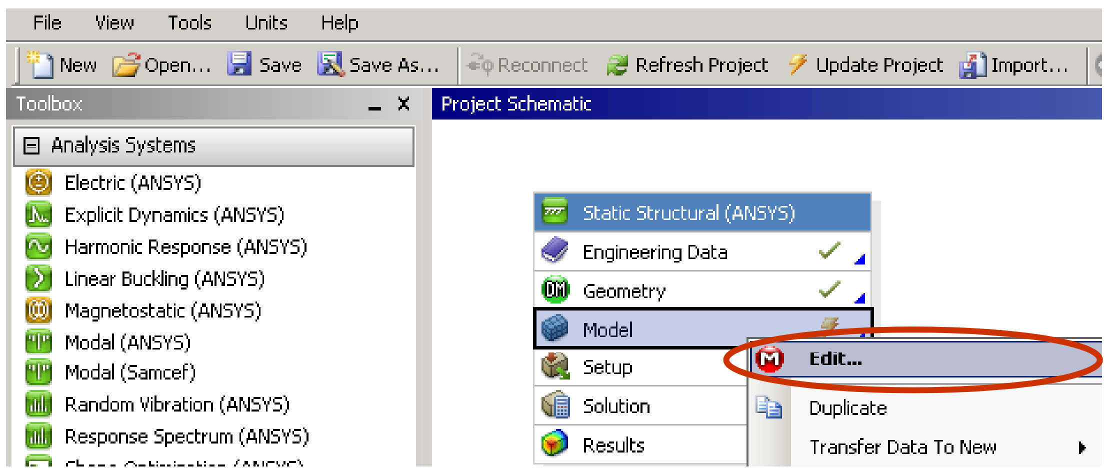


- **Open the Engineering Data Cell (highlight and double click OR Right Mouse Button (RMB)>Edit) to verify the linear material properties.**
- **Verify that the units are in Metric(Tonne,mm,…) system. If not, fix this by clicking on…**
  - **Utility Menu > Units > Metric(Tonne, mm,…)**



… Workshop 5A – Metal Plasticity

- **Double click on the Model Cell to open the FE Model (Mechanical Session) (or RMB=>Edit…)**



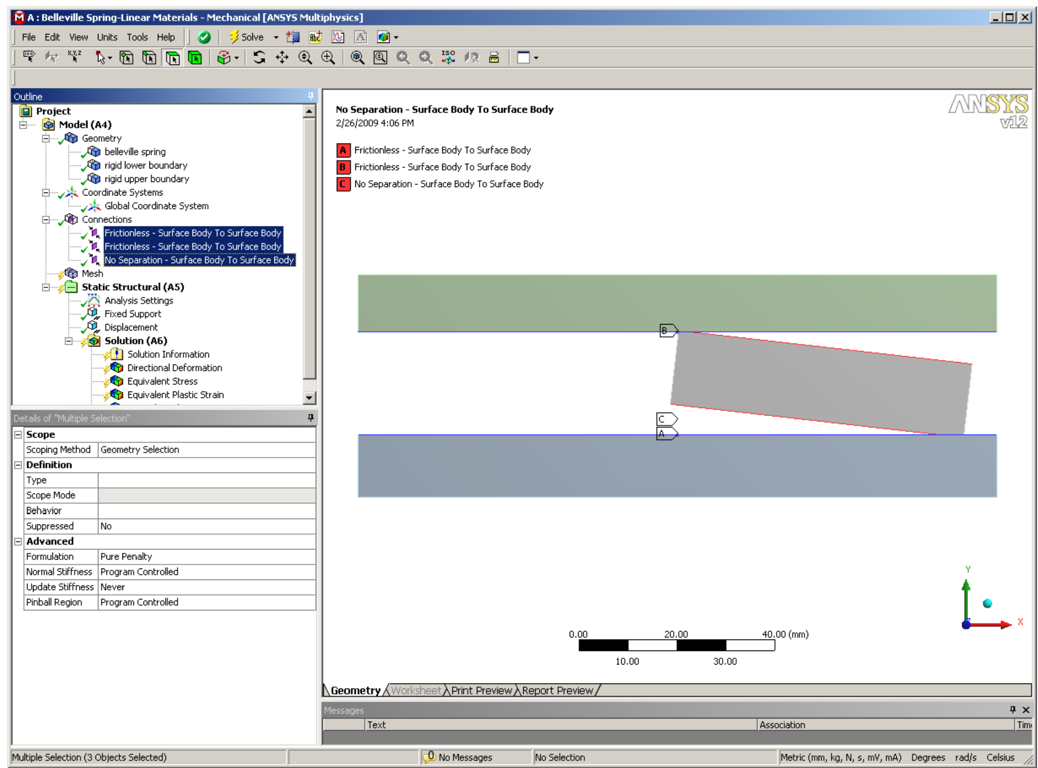
… Workshop 5A – Metal Plasticity

- **Once inside the Mechanical application, verify the working unit system**
  - **"Unit > Metric (mm,kg,N,s,mV,mA)"**
- **Open the folders beneath the model branch to become familiar with the model set-up.**

Highlight "Geometry" and refer to the details window to verify that this is a 2D axisymmetric model.

Inspect the two asymmetric frictionless contact regions on top and bottom of spring which interface with top and bottom rigid boundaries.

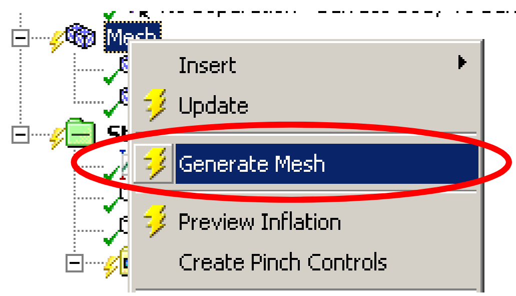
Inspect the no-separation contact region which ties down the spring at the bottom corner to prevent rigid body motion during unloading.

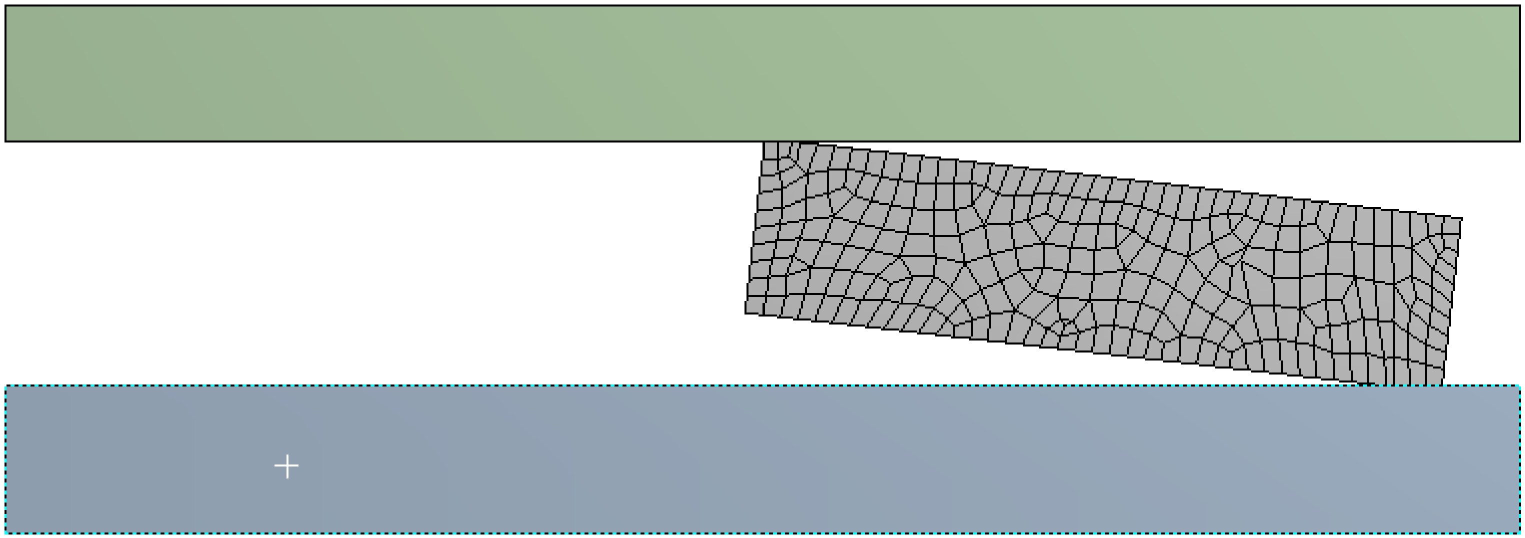


… Workshop 5A – Metal Plasticity

- **Review the mesh:**
  - **RMB>Generate Mesh**

The upper and lower geometries are meshed with one element each, while the belleville spring geometry is a free mesh.





… Workshop 5A – Metal Plasticity

This is going to be a 3 load step analysis:

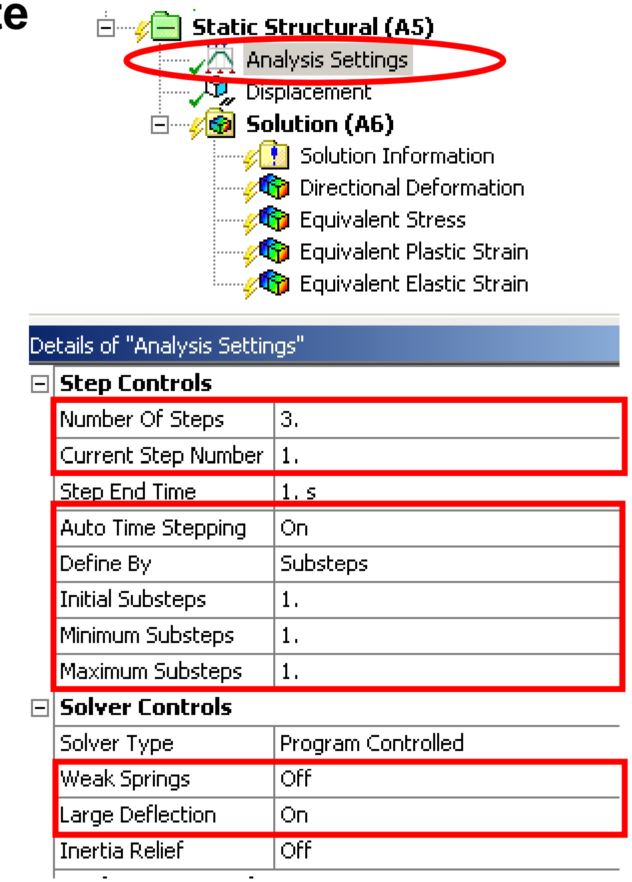
With the bottom plate fixed:
LS1: Null Solution (to generate results at origin for force-deflection plot)
LS2: Apply displacement load (-5mm) to upper plate
LS3: Remove displacement load

- **Confirm the following Analysis Settings:**

Number of Steps:	***3***
Weak Springs:	***Off***
Large Deflection:	***On***

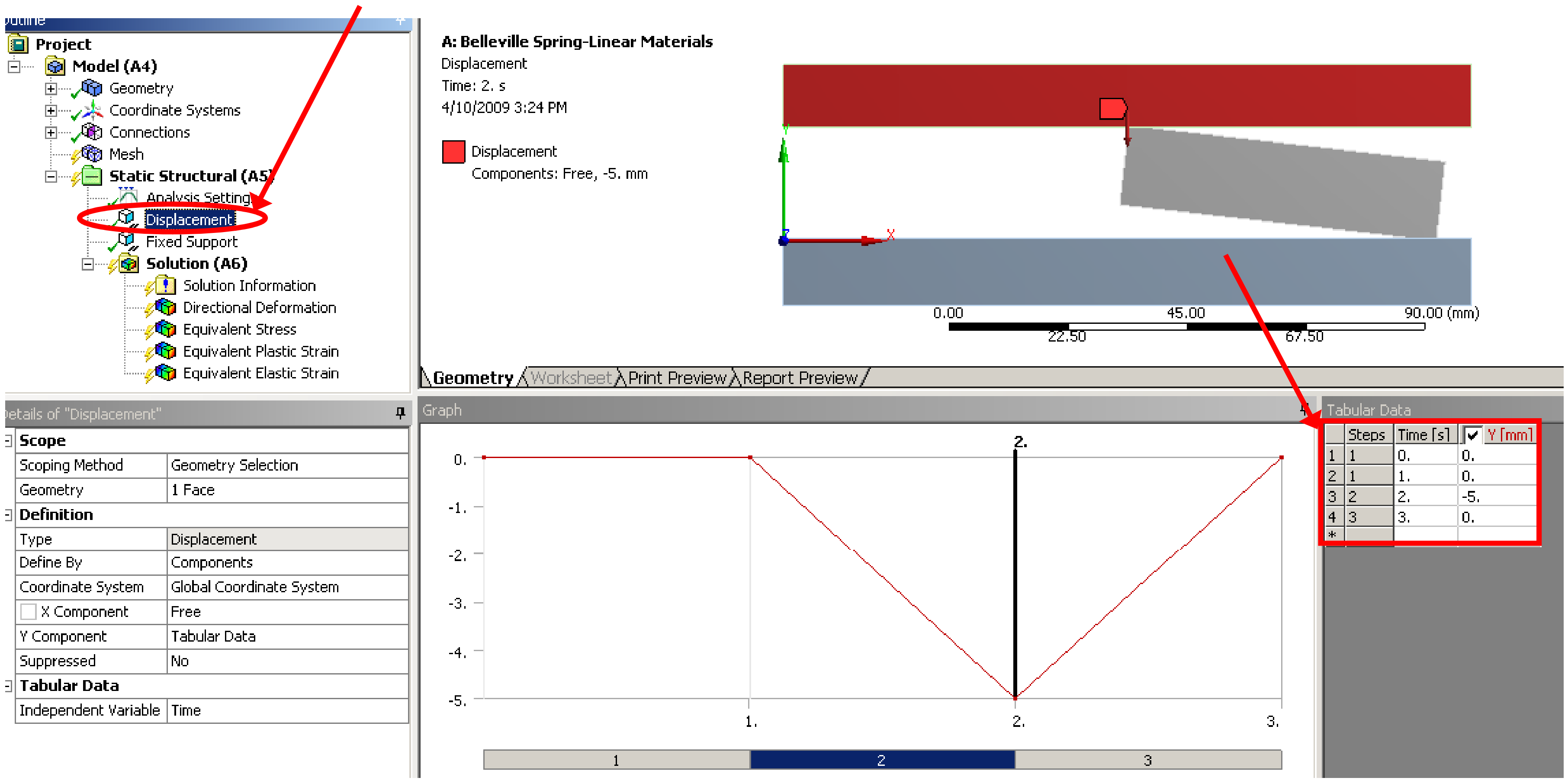
For Current Step Number =1, Auto Time Stepping *On and with Initial, Minimum and Maximum Substeps = '1'.* (Null Solution)

For Current Step Numbers 2 and 3, Program Controlled for Auto Time Stepping.



… Workshop 5A – Metal Plasticity

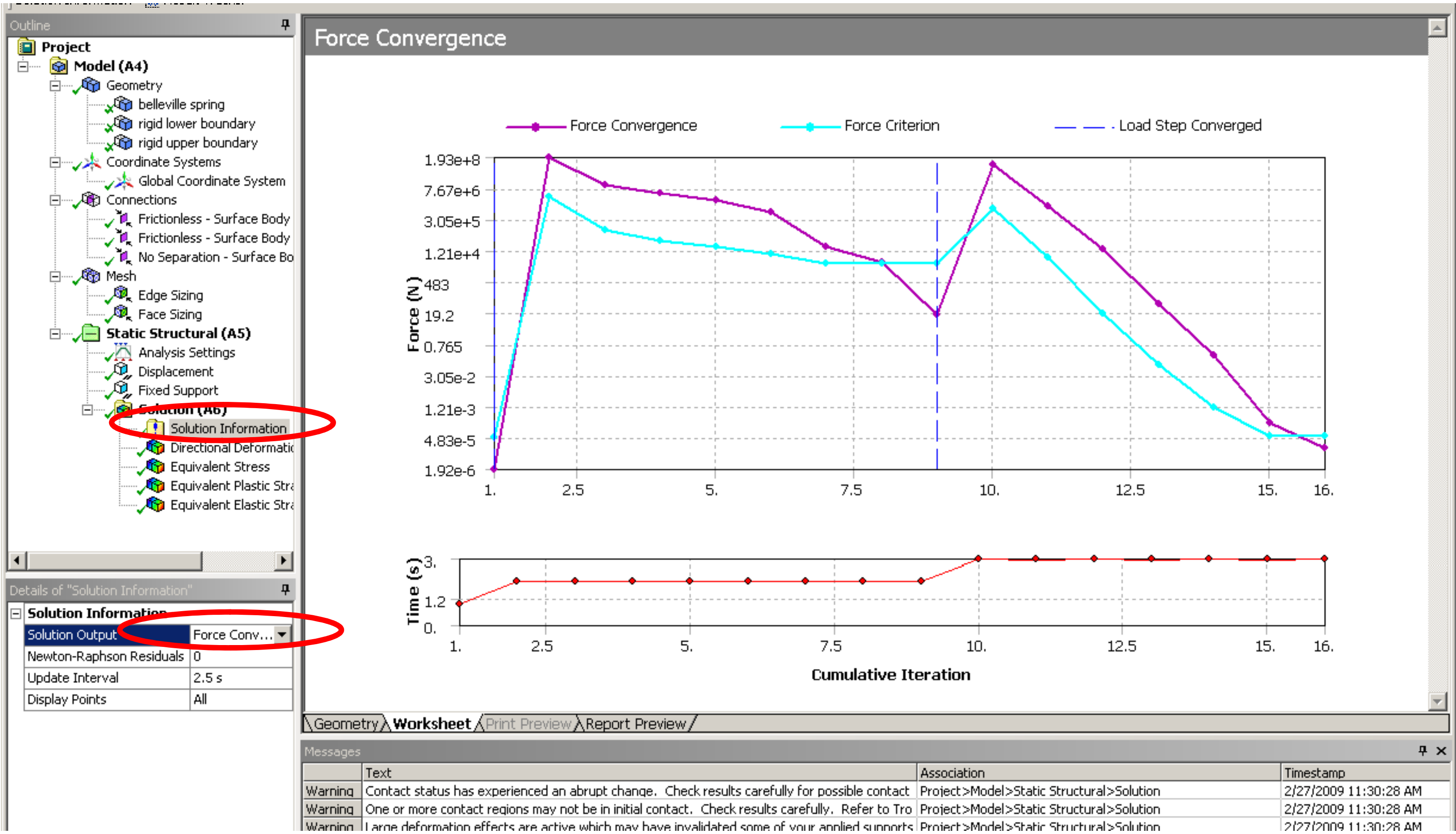
- **Review the predefined displacement load on the upper plate for the three load steps.**



… Workshop 5A – Metal Plasticity

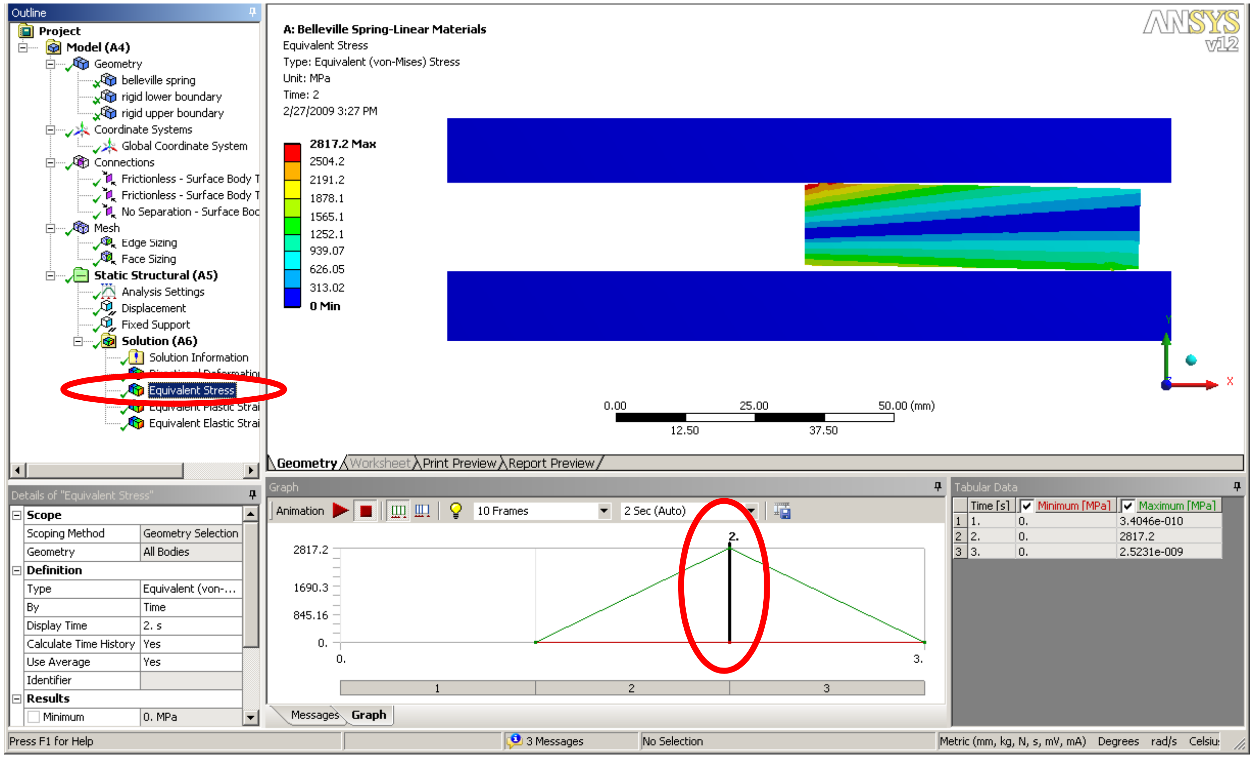
- **Execute Solve:** 

- **After solution is complete, review convergence history:**



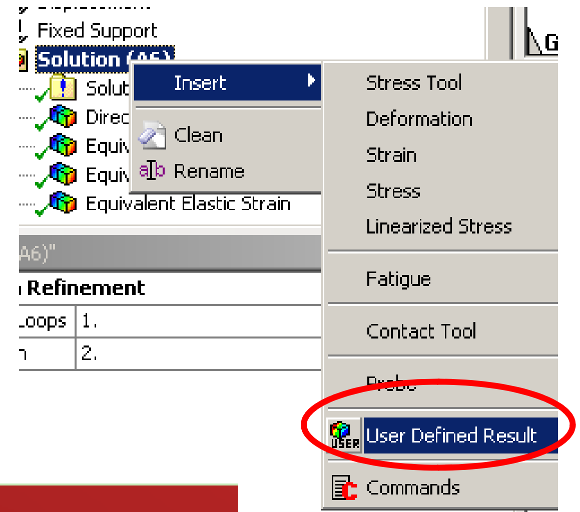
… Workshop 5A – Metal Plasticity

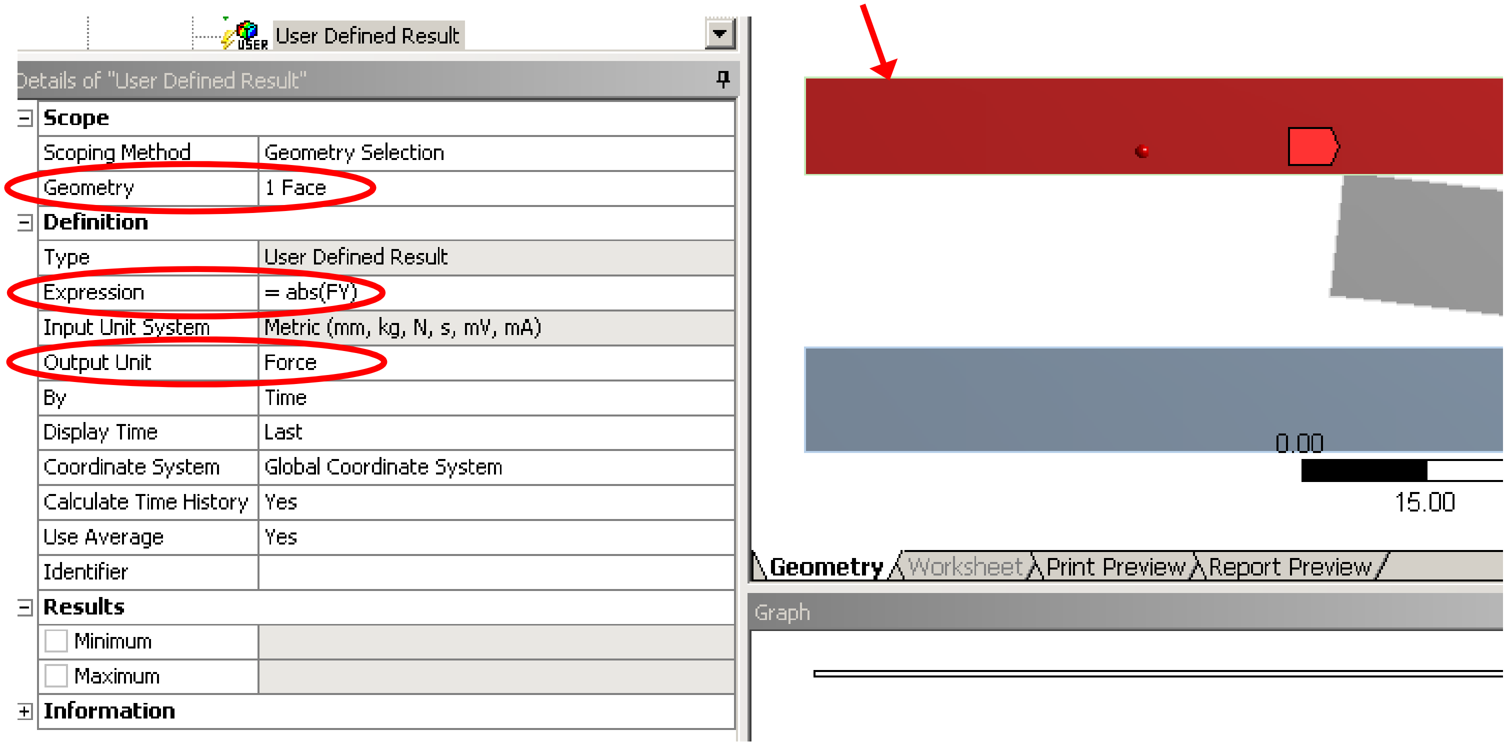
- **Post Process results at Load step 2:**
  - **Note how high the stress in the spring is at the end of LS2. Recall, this is still linear elastic material.**
  - **At LS3 (not shown), plastic strain is zero and there is no permanent deformation of the spring upon unloading as expected.**



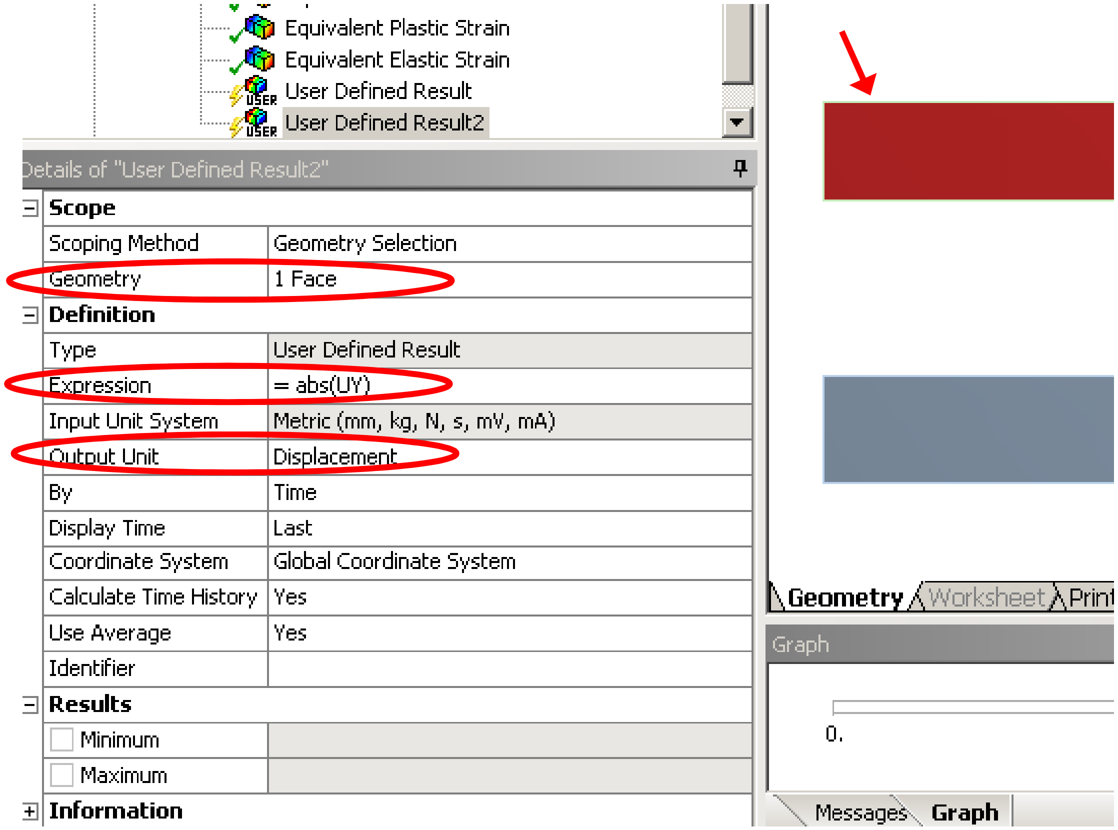
… Workshop 5A – Metal Plasticity

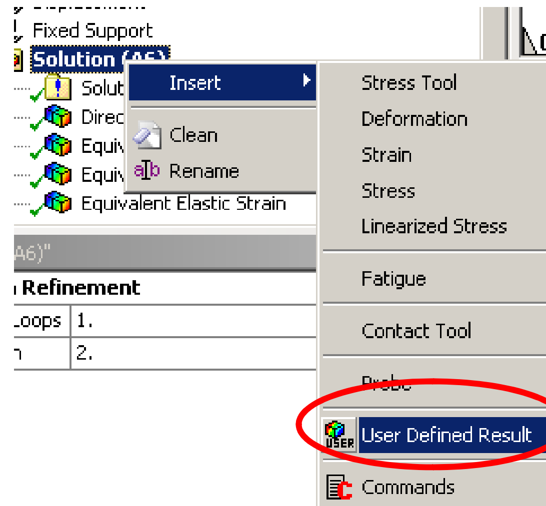
- **Generate Force vs. Deflection Curve of Spring**
  - **With Solution Branch Highlighted: RMB>Insert>User Defined Result**
  - **Scope result to the upper rigid plate**
  - **Define the expression as ‘abs(FY)’ for absolute value of force in Y-direction**



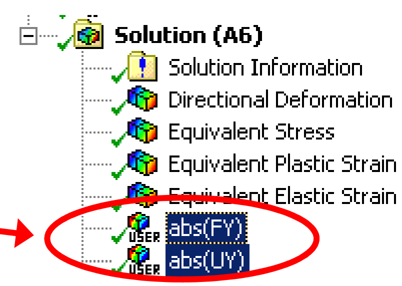


Repeat Procedure for Displacement in ‘UY’



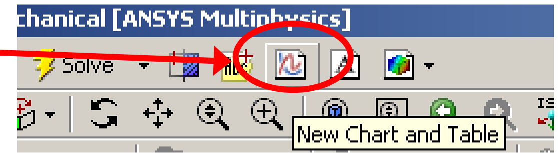


- Highlight both User Defined Results: RMB>Rename based on Definition
- Highlight Solution Branch: RMB>Evaluate results

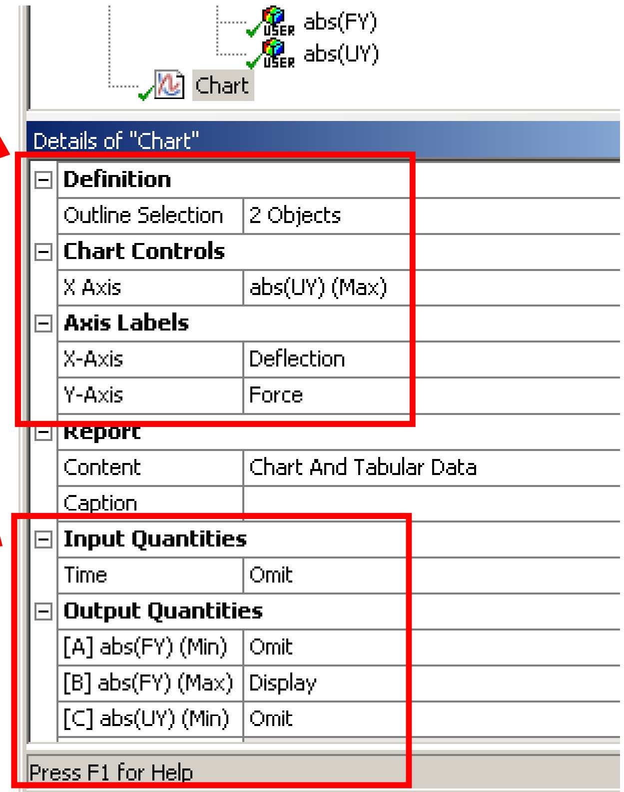


… Workshop 5A – Metal Plasticity

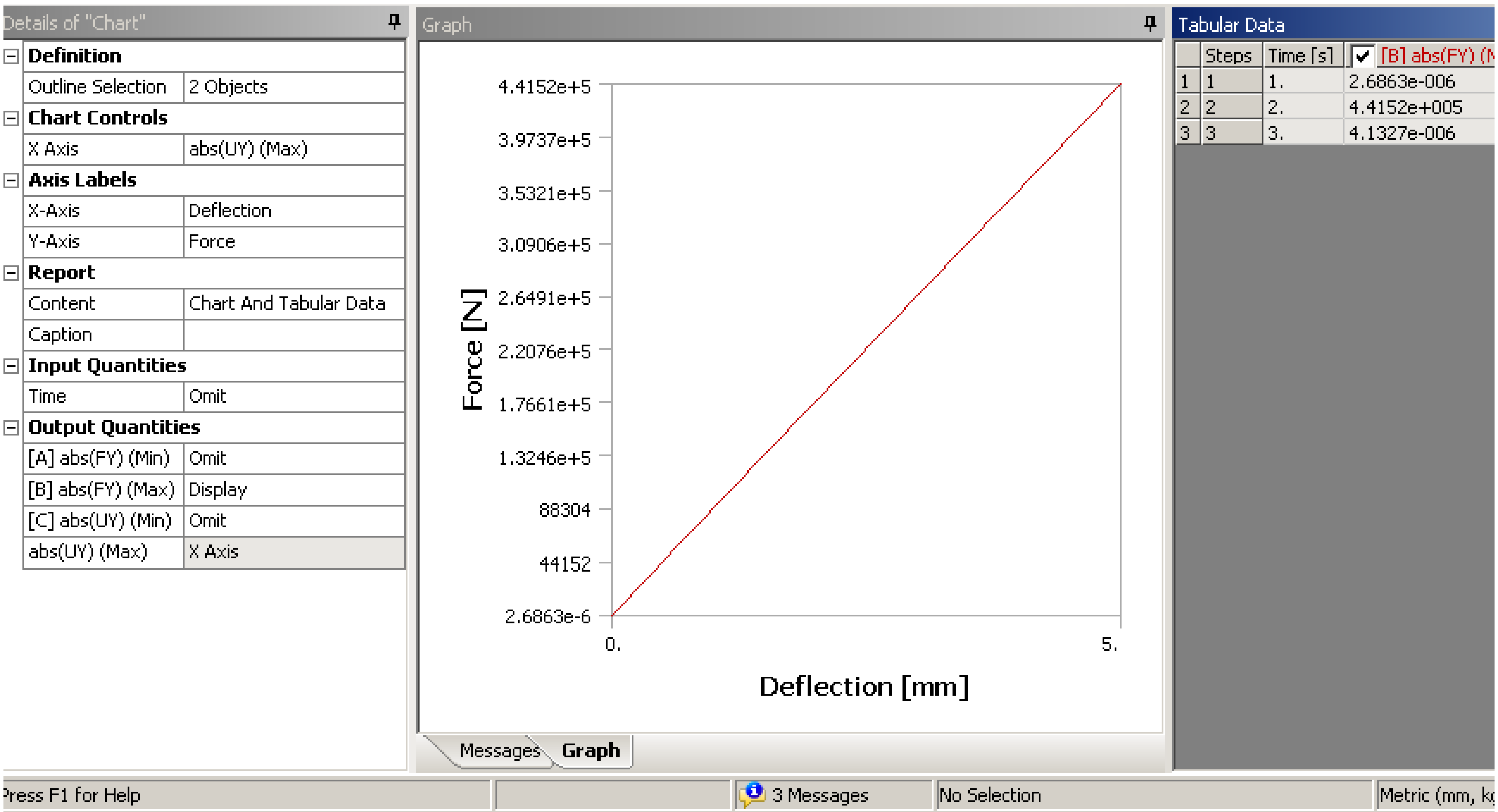
- Insert a Chart Tool for plotting FY vs UY



  - Fill in Chart tool Details Window as Follows:
    - Outline Selection: Select 'abs(FY)' and 'abs(UY)' from Solution Branch
    - X Axis: abs(UY) (Max)
    - X-Axis Label: Deflection
    - Y-Axis Label: Force
    - Omit: Time, abs(FY)(Min), abs(UY)(Min)

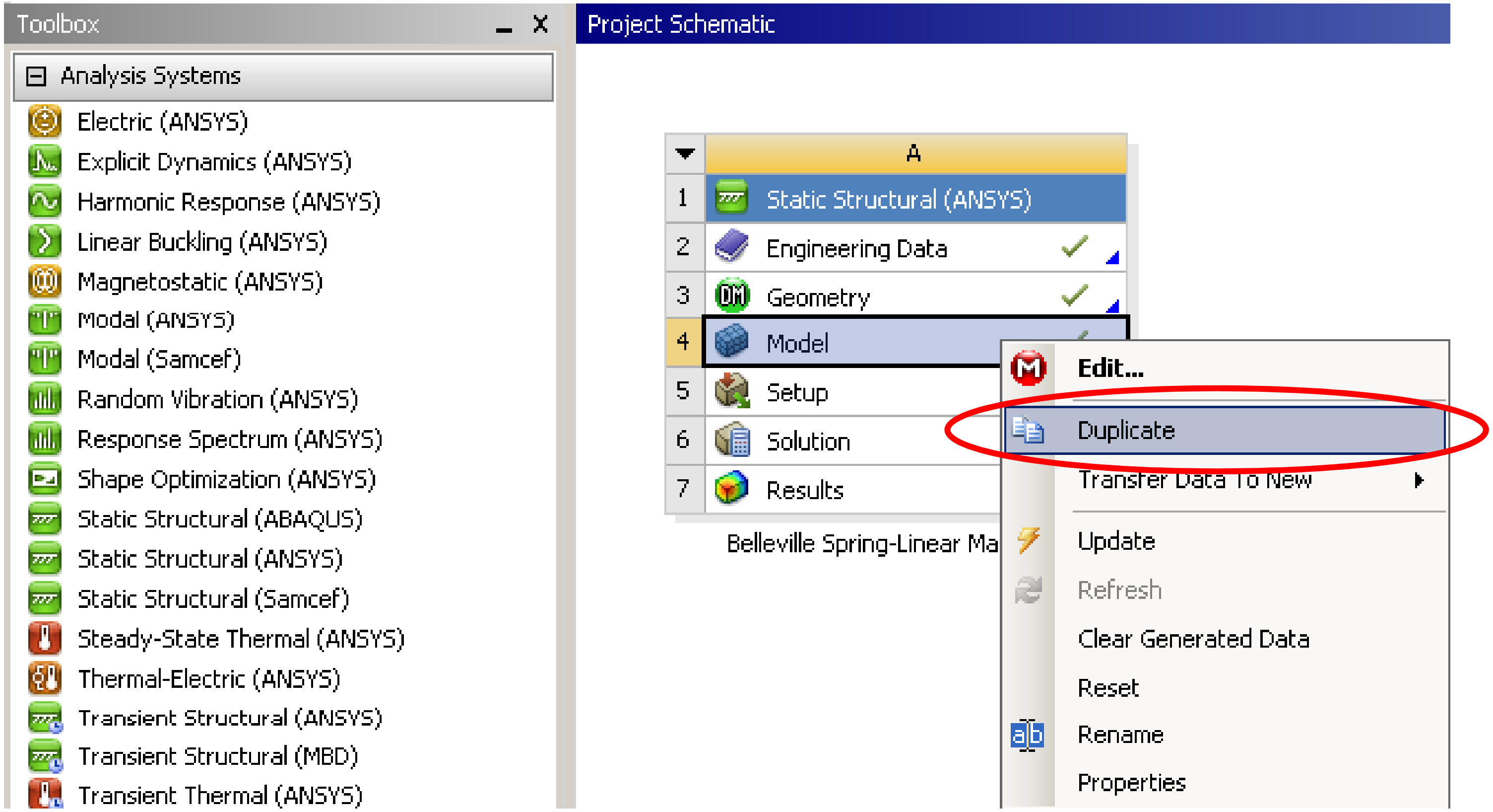


Resulting Chart of Force vs Displacement for linear material is a straight line with no permanent deformation as expected



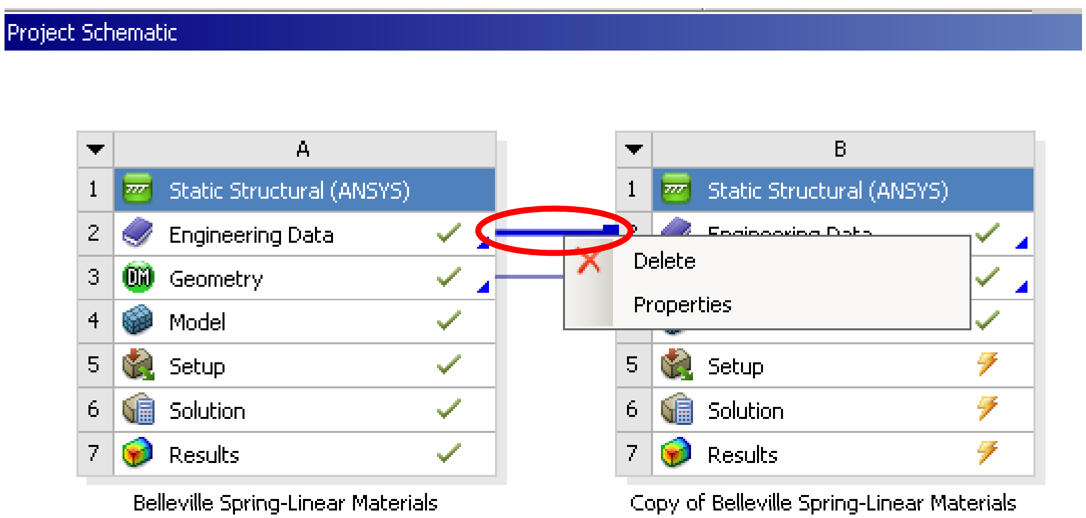
… Workshop 5A – Metal Plasticity

- **Duplicate the Static Analysis**
  - **Return to the Project Schematic**
  - **Highlight the Model Cell and RMB> Duplicate**



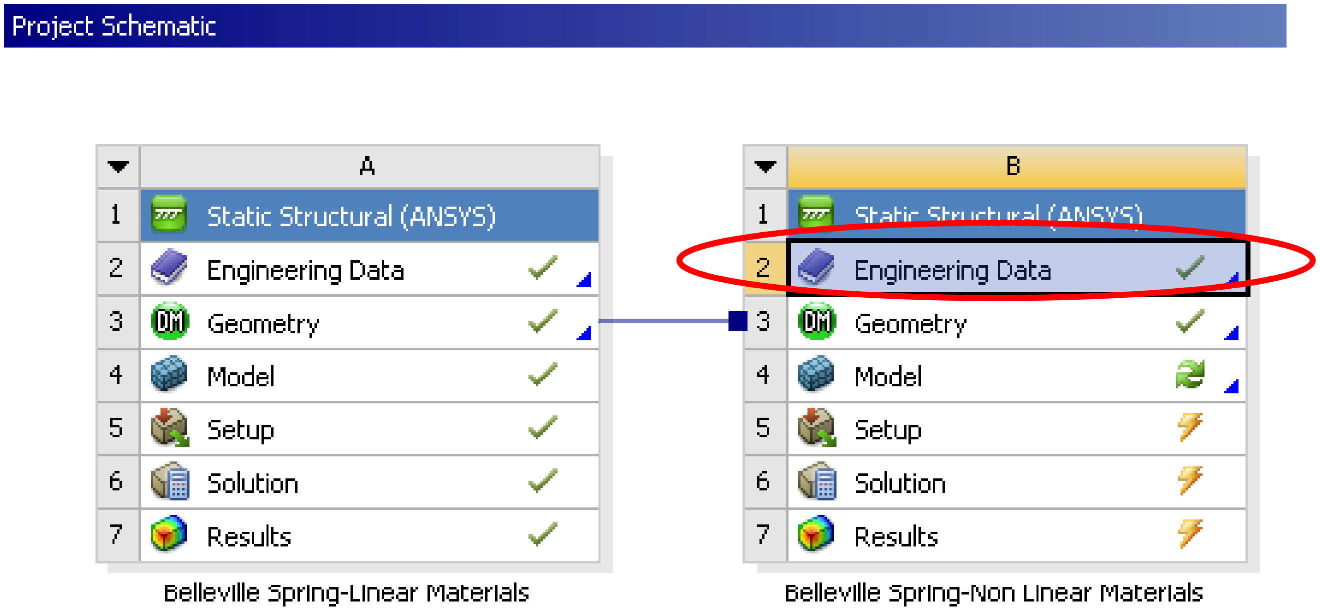
… Workshop 5A – Metal Plasticity

- **Disassociate material properties link**
  - **The second analysis is going to be with metal plasticity defined**
  - **Highlight the Engineering data link and RMB>Delete**



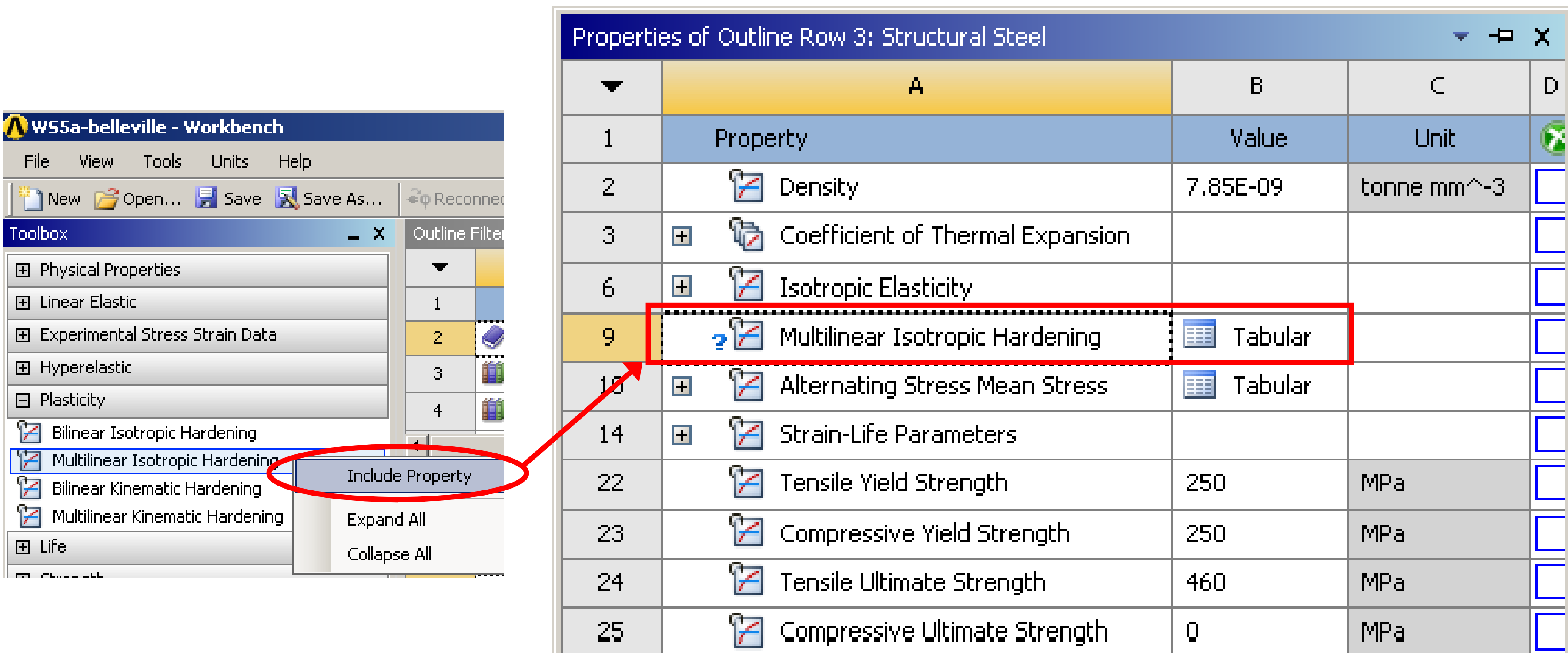
… Workshop 5A – Metal Plasticity

- **Project Schematic should now look like the diagram below**
  - **We can now modify the Engineering data in Table B without effecting the model and/or results of Table A.**
  - **Change the title of the new Analysis to:**
    "Belleville Spring-Nonlinear Materials"
  - **Open the Engineering Data Cell in Table B**



… Workshop 5A – Metal Plasticity

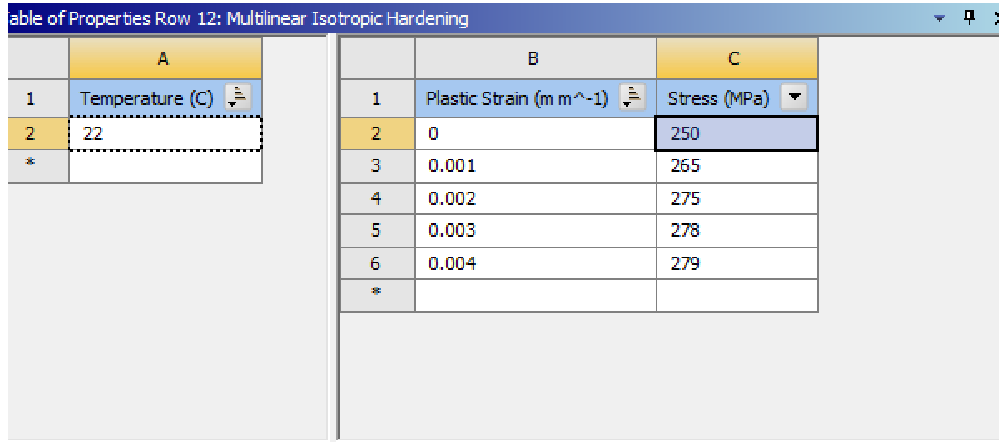
- **Insert a Metal Plasticity Model**
  - **From the Tool Box, open the Plasticity Folder**
  - **Highlight Multilinear Isotropic Hardening and RMB>"Include Property"**
  - **The new material should now appear in the Properties dialogue box**



… Workshop 5A – Metal Plasticity

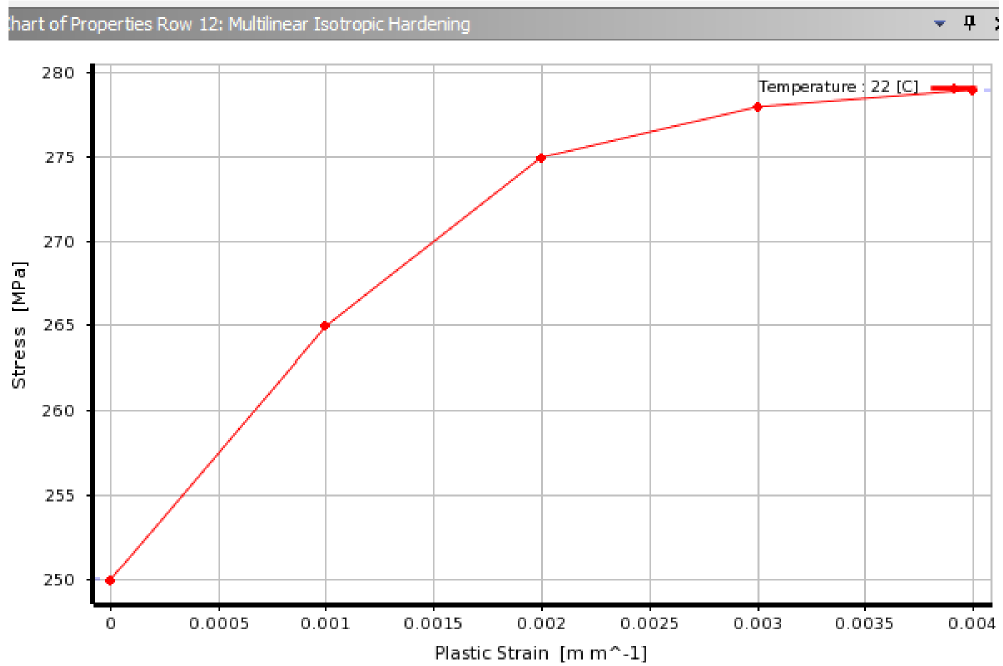
- **Define Plasticity data**
  - **Fill in plastic strain and stress data as shown to the right**
  - **From the Utility Menu, read in the modified material properties with**
    - **Refresh Project**

  - **Return to Project Workspace**



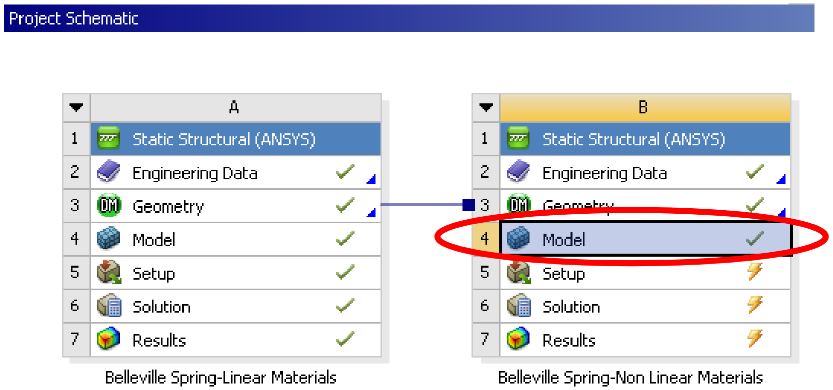
	A
1	Temperature (C)
2	22
*	

	B	C
1	Plastic Strain (m m^-1)	Stress (MPa)
2	0	250
3	0.001	265
4	0.002	275
5	0.003	278
6	0.004	279
*		

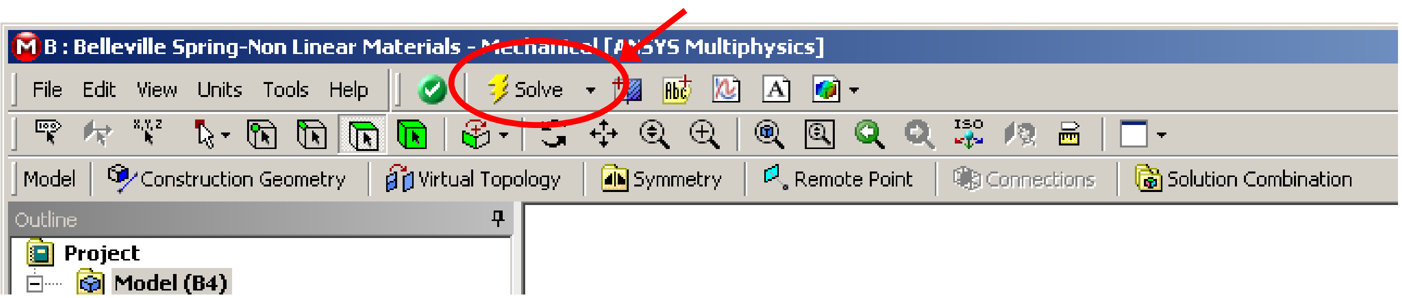


… Workshop 5A – Metal Plasticity

- **From the project schematic, highlight and open the model cell in Table B.**



- **All the geometry entities, meshing specs, boundary conditions, loads and analysis settings are preserved from the previous analysis.**
  - **Execute the Solve with the newly defined plasticity properties**



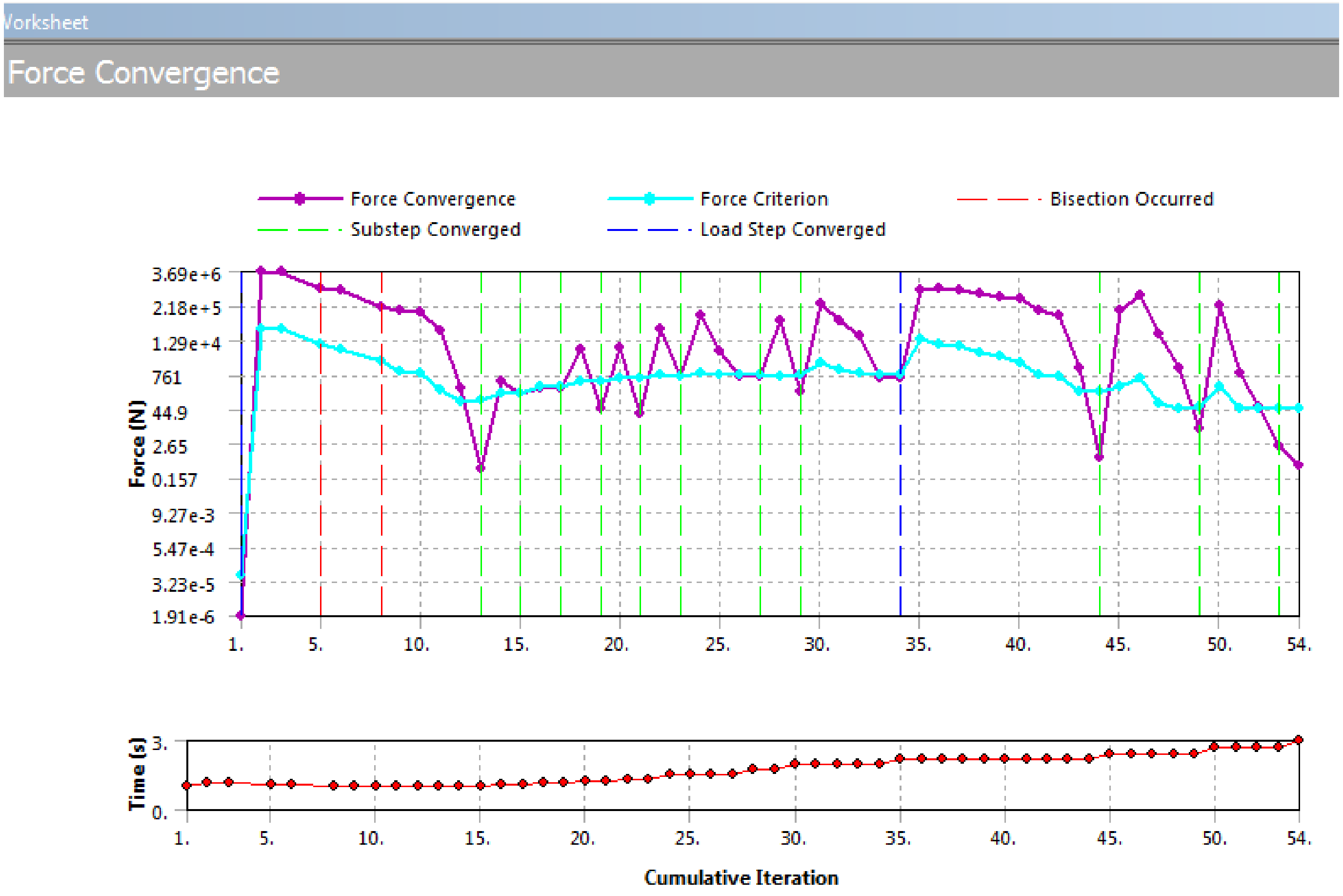
… Workshop 5A – Metal Plasticity

- **After solution is complete, review Solution output:**
  - **Confirm that the metal plasticity, as defined, was included in this new run**

```
PLASTIC  (PLAS) Table For Material     3

  ISOTROPIC HARDENING PLASTICITY

       Temperature = 22.000000

   Point        PlStrain        Stress
       1     0.000000e+000   2.500000e+002
       2     1.000000e-003   2.650000e+002
       3     2.000000e-003   2.750000e+002
       4     3.000000e-003   2.780000e+002
       5     4.000000e-003   2.790000e+002


   *** LOAD STEP     2   SUBSTEP     3  COMPLETED.    CUM ITER =     16
   *** TIME =   1.08575          TIME INC =  0.367500E-01
   *** MAX PLASTIC STRAIN STEP = 0.4841E-04   CRITERION = 0.1500
   *** AUTO TIME STEP:  NEXT TIME INC = 0.55125E-01  INCREASED (FACTOR = 1.5000)
```

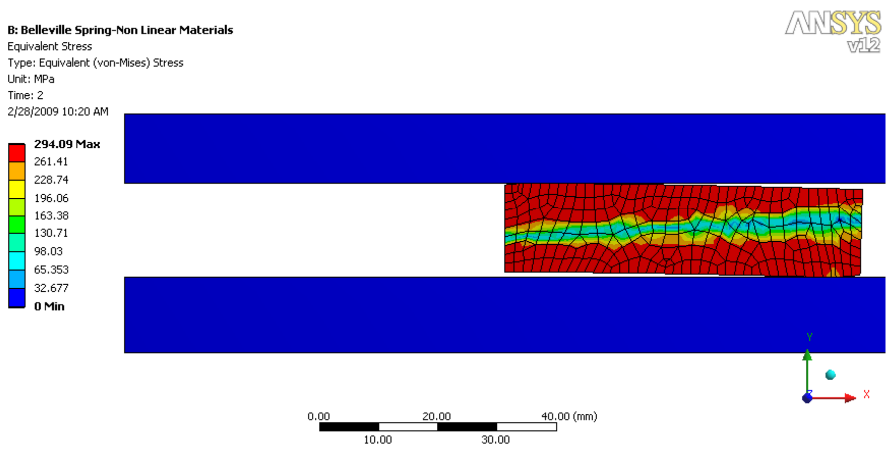
… Workshop 5A – Metal Plasticity

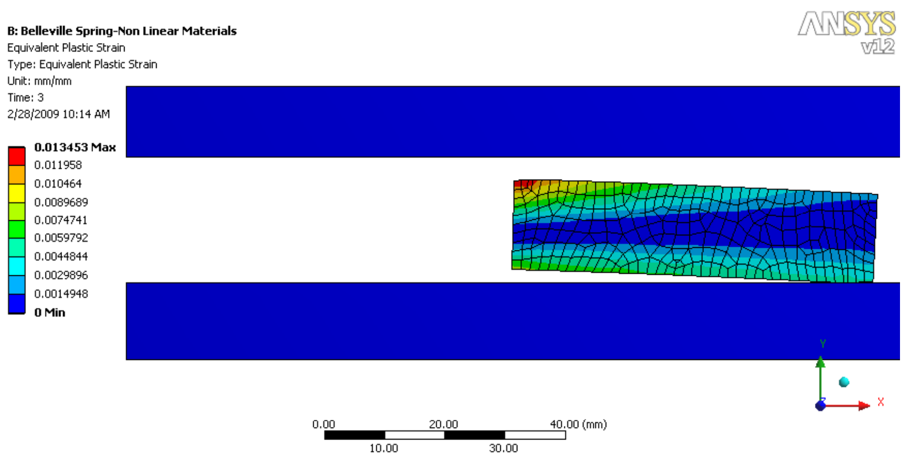
- **Review the Convergence History. Compare this with the Linear material run.**



… Workshop 5A – Metal Plasticity

- **Post Process results at Load step 2 as before:**
  - **Compare the max stress in this material with the linear material**
  - **Note also that the spring now takes a permanent set after load is removed as expected.**





… Workshop 5A – Metal Plasticity

- Highlight the Chart tool and Plot Force vs Deflection as before.
  - Note the nonlinear path of the curve reflecting the influence of the material yielding and taking a permanent set.
  - Note also the difference in the magnitude of the load required to produce the same deflection with this material verses the linear material, underscoring the importance of considering nonlinear material behavior in some designs.

Details of "Chart"

Definition	
Outline Selection	2 Objects
Chart Controls	
X Axis	abs(UY) (Max)
Axis Labels	
X-Axis	Deflection
Y-Axis	Force
Report	
Content	Chart And Tabular Data
Caption	
Input Quantities	
Time	Omit
Output Quantities	
[A] abs(FY) (Min)	Omit
[B] abs(FY) (Max)	Display
[C] abs(UY) (Min)	Omit
abs(UY) (Max)	X Axis